Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.1-Sine/75-4.1.2.3-g-sin-^p-a+b-sin-^m-c+d-sin-^n

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [51]. This is test number [75].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (51)	0.00 (0)
Maple	98.04 (50)	1.96 (1)
Mathematica	92.16 (47)	7.84 (4)
Fricas	60.78 (31)	39.22 (20)
Giac	39.22 (20)	60.78 (31)
Maxima	31.37 (16)	68.63 (35)
Mupad	25.49 (13)	74.51 (38)
Sympy	7.84 (4)	92.16 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

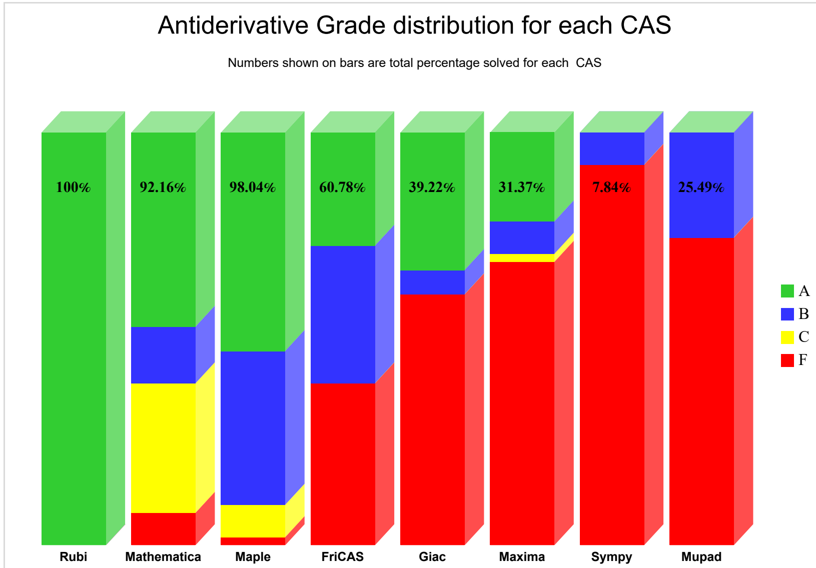
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

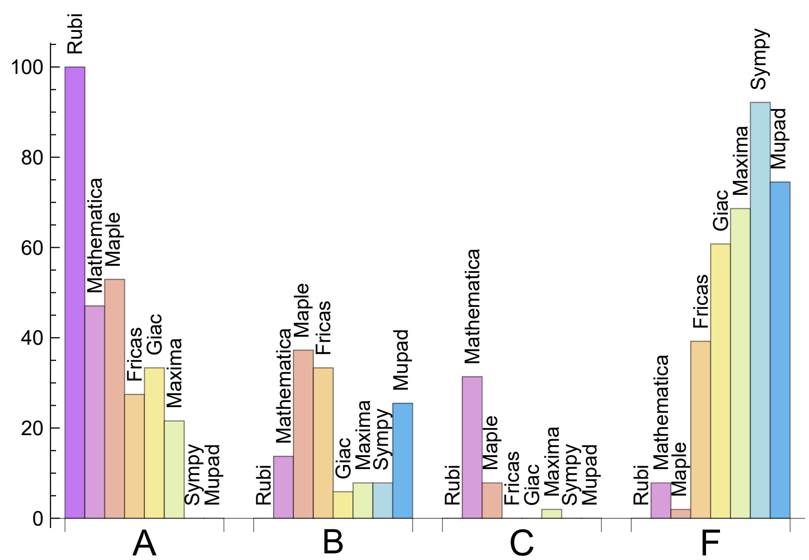
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	52.941	37.255	7.843	1.961
Mathematica	47.059	13.725	31.373	7.843
Giac	33.333	5.882	0.000	60.784
Fricas	27.451	33.333	0.000	39.216
Maxima	21.569	7.843	1.961	68.627
Mupad	0.000	25.490	0.000	74.510
Sympy	0.000	7.843	0.000	92.157

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	40.00	60.00	0.00
Giac	31	67.74	25.81	6.45
Maxima	35	97.14	0.00	2.86
Mupad	38	0.00	100.00	0.00
Sympy	47	89.36	10.64	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Sympy	0.24
Maxima	0.24
Rubi	0.25
Giac	0.37
Maple	2.14
Fricas	2.62
Mathematica	7.78
Mupad	13.99

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	124.85	1.35	114.50	1.30
Maxima	142.81	1.98	121.50	1.56
Rubi	146.39	1.01	121.00	1.00
Sympy	273.50	3.08	273.00	3.16
Fricas	952.00	7.60	240.00	3.31
Mathematica	1794.38	8.26	203.00	1.57
Mupad	2002.92	12.16	163.00	2.21
Maple	7015.90	22.79	225.00	1.86

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

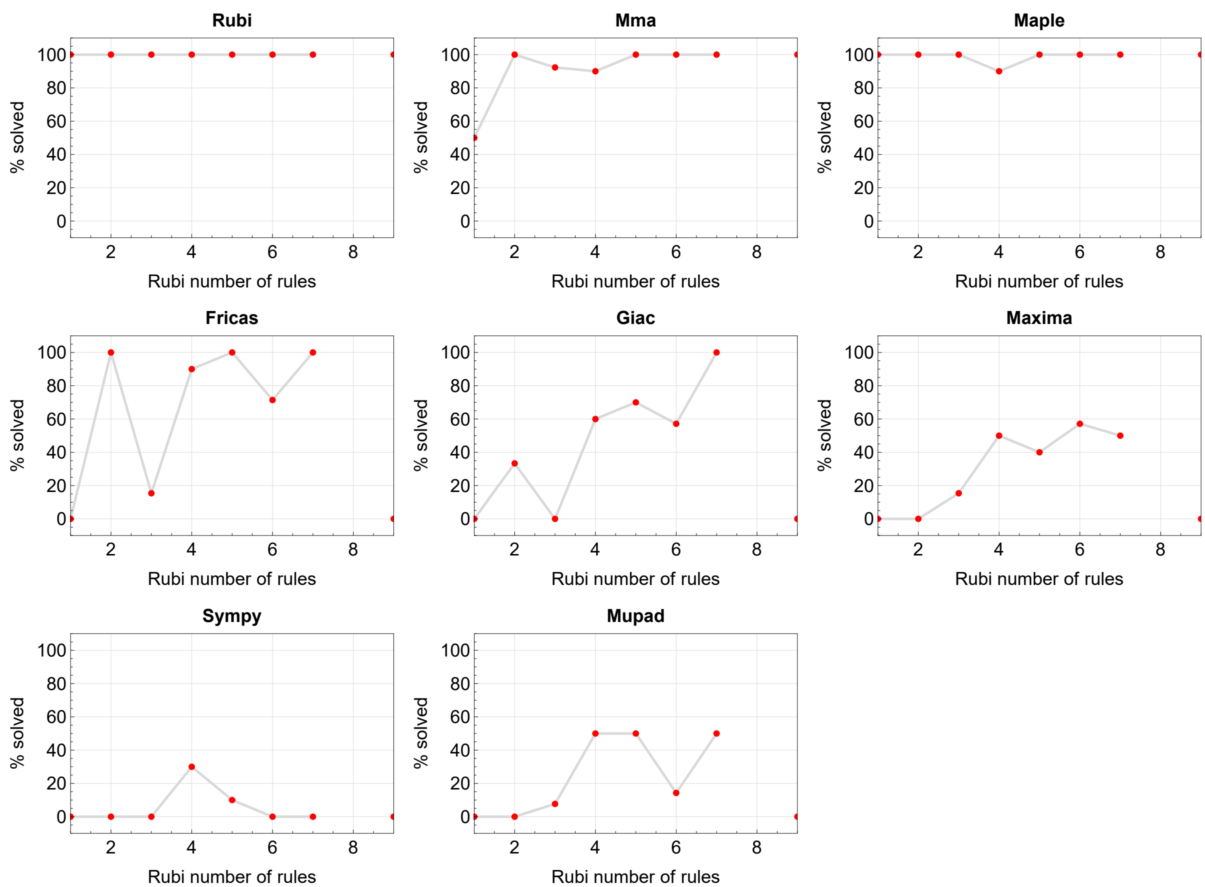


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

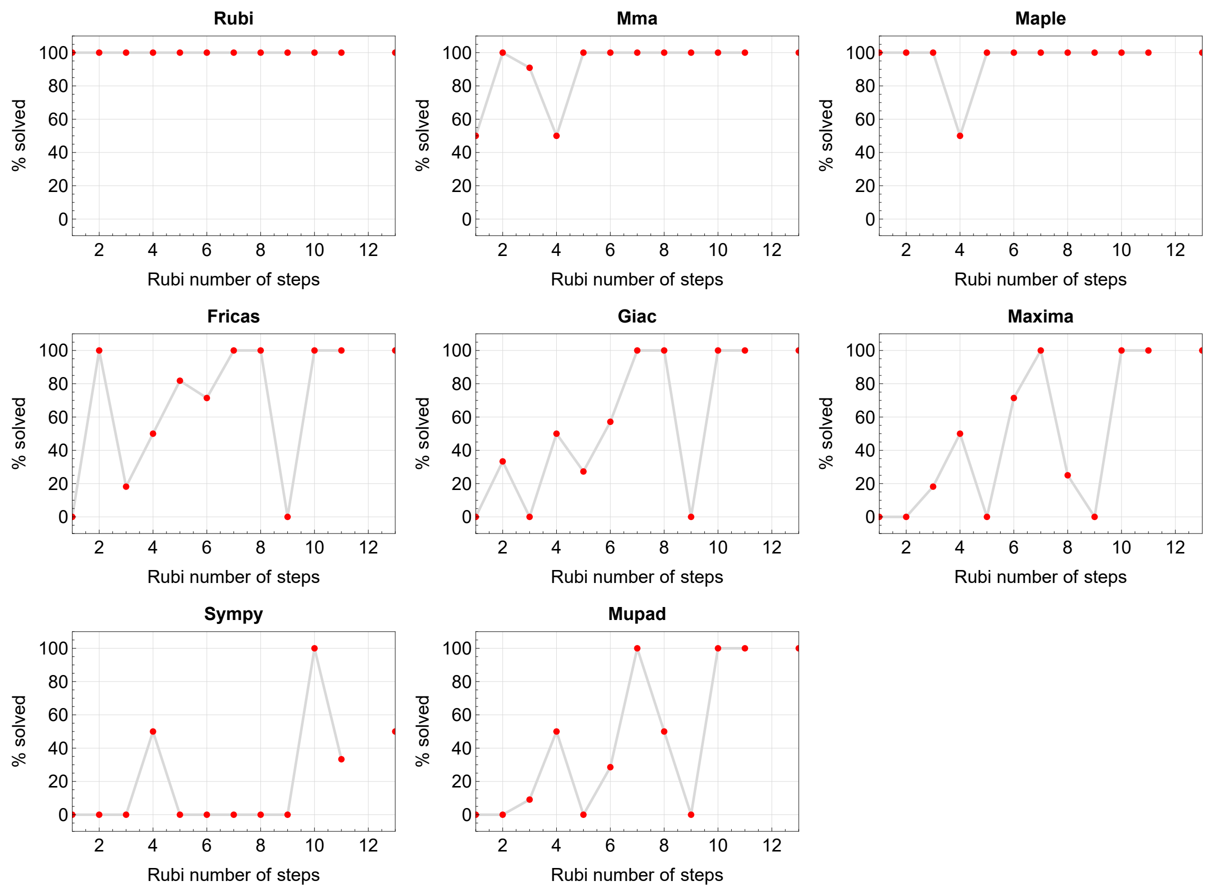


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

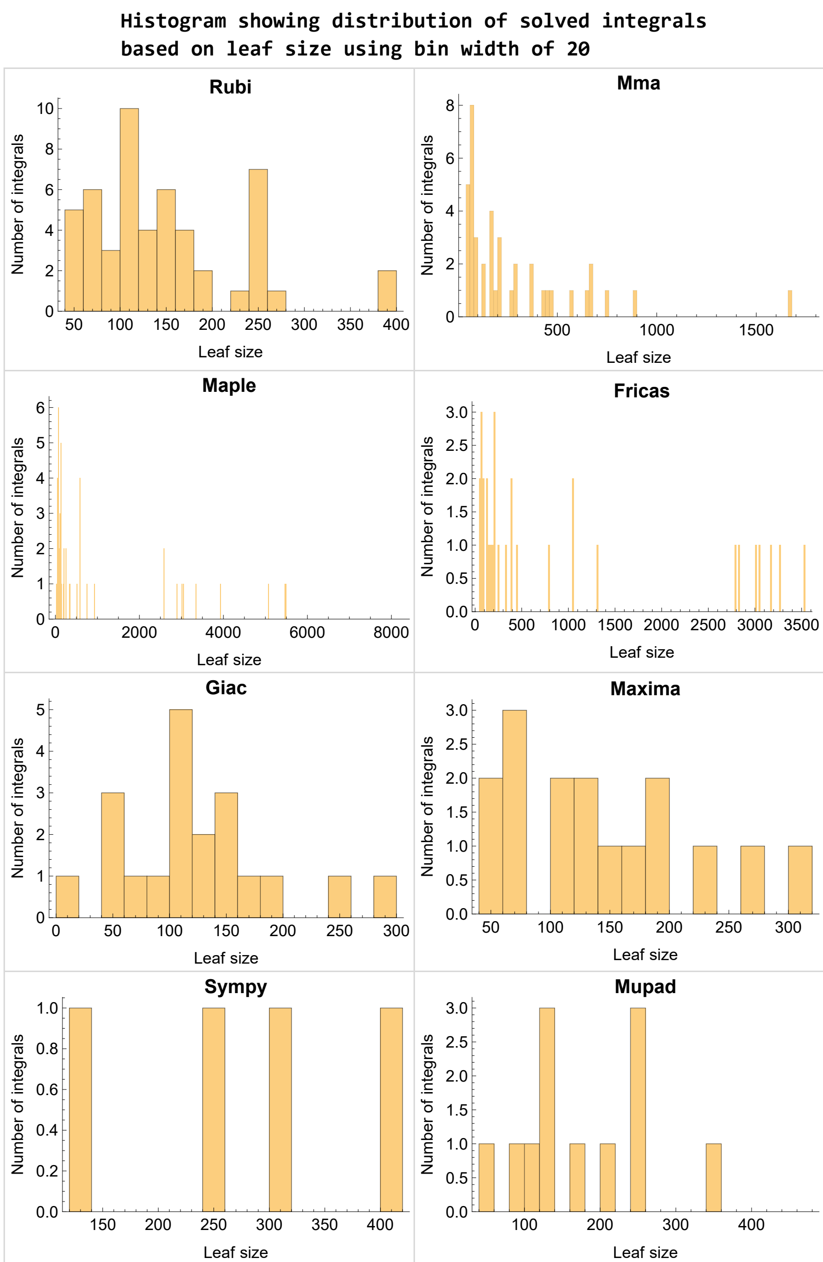


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

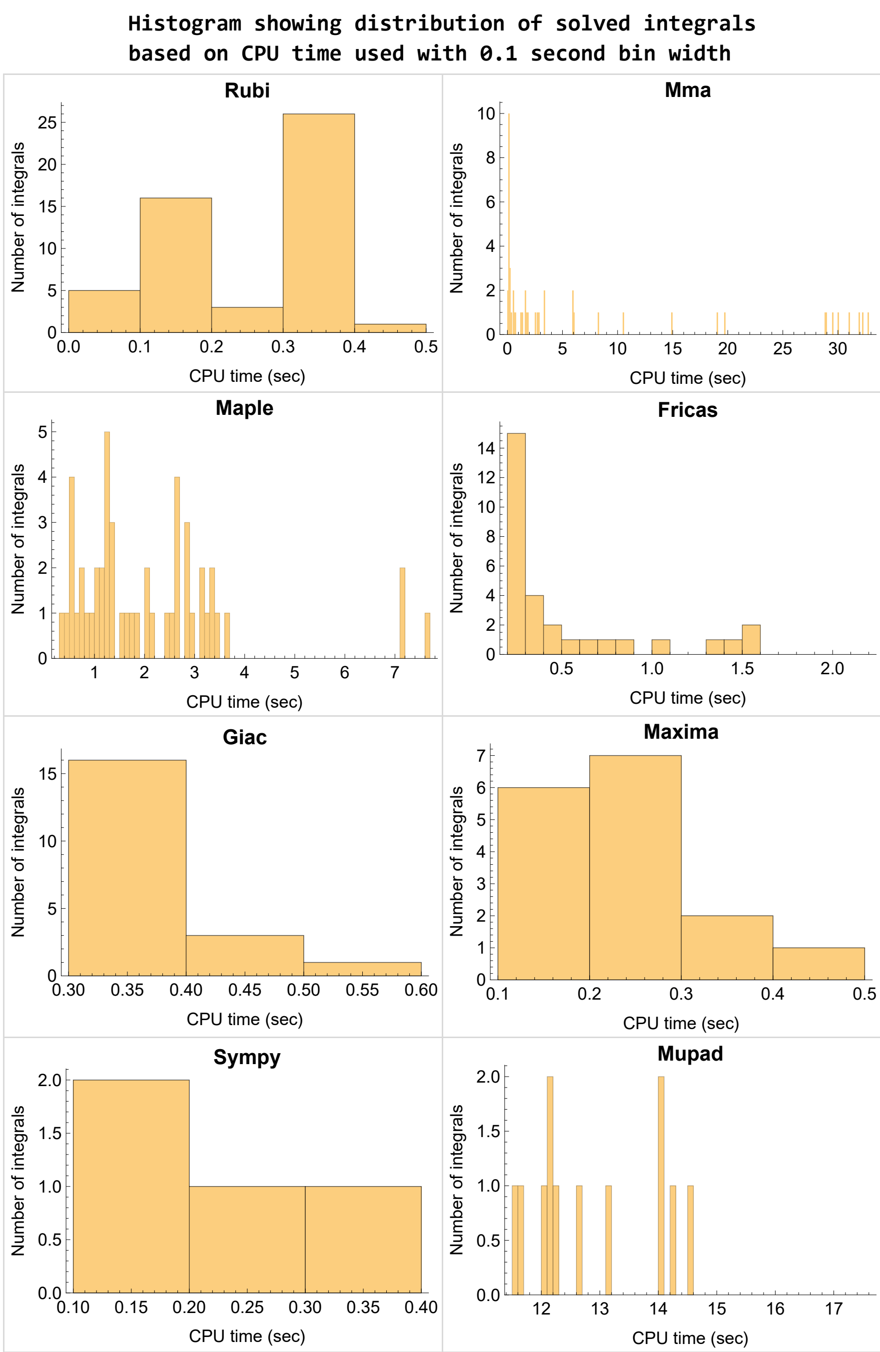


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

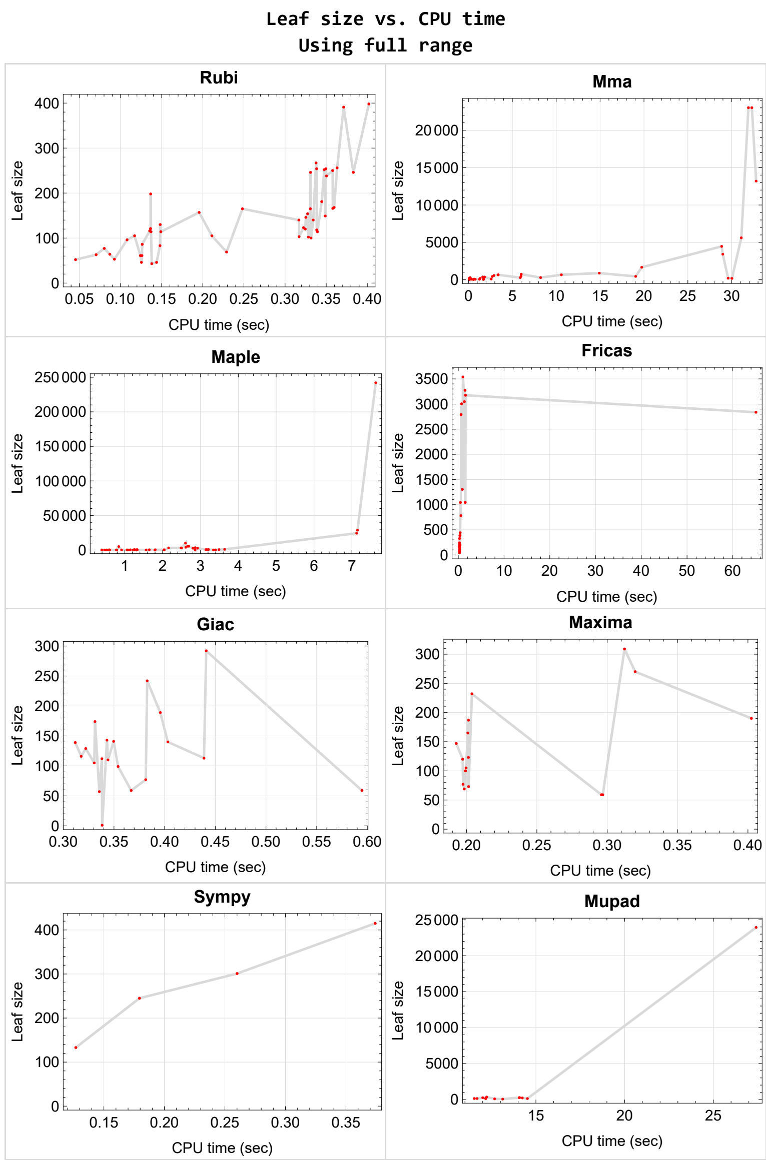


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{26, 31, 32, 33, 34, 35, 36, 37, 38, 42, 45, 46\}$

Maple $\{15, 25, 26, 27, 28, 31, 32, 33, 34, 35, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

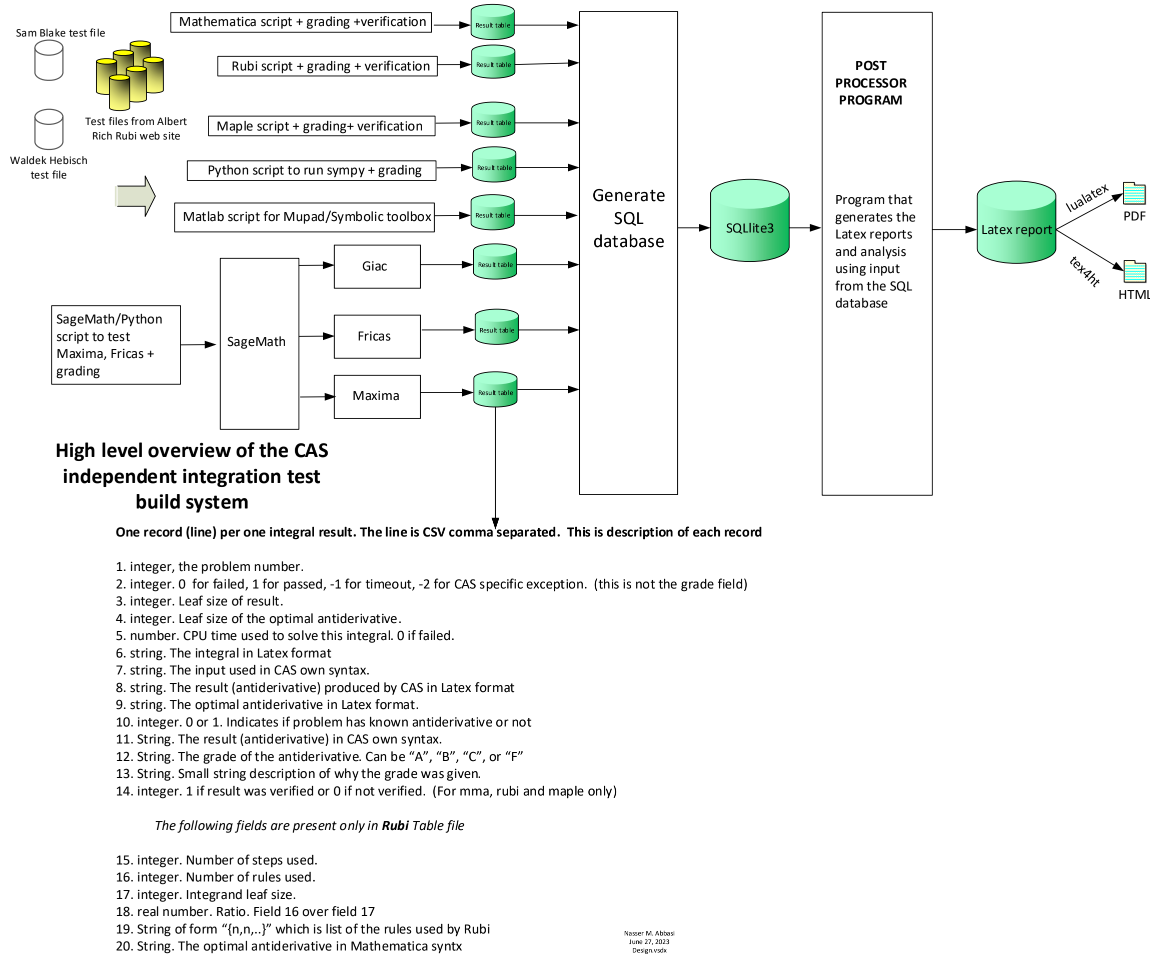
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 36

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 39, 48, 49, 50 }

B grade { 8, 9, 32, 33, 37, 38, 45 }

C grade { 13, 14, 23, 24, 25, 26, 29, 30, 31, 34, 35, 36, 40, 41, 42, 46 }

F normal fail { 43, 44, 47, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 29, 30, 39, 40, 41 }

B grade { 15, 22, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 47, 49, 50 }

C grade { 31, 42, 46, 48 }

F normal fail { 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 12, 15, 16, 17, 18, 19, 22, 27 }

B grade { 7, 8, 9, 10, 11, 13, 14, 23, 24, 25, 26, 28, 35, 36, 37, 38, 39 }

C grade { }

F normal fail { 20, 21, 31, 32, 33, 34, 45, 51 }

F(-1) timedout fail { 29, 30, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 20, 21 }

B grade { 8, 9, 16, 17 }

C grade { 22 }

F normal fail { 12, 13, 14, 15, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { 39 }

Giac

A grade $\{ 1, 2, 3, 4, 5, 7, 9, 10, 12, 13, 14, 19, 20, 21, 23, 24, 39 \}$

B grade $\{ 6, 8, 11 \}$

C grade $\{ \}$

F normal fail $\{ 16, 26, 29, 30, 31, 32, 33, 34, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 \}$

F(-1) timedout fail $\{ 15, 17, 18, 25, 27, 35, 37, 38 \}$

F(-2) exception fail $\{ 22, 28 \}$

Mupad

A grade $\{ \}$

B grade $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 39 \}$

C grade $\{ \}$

F normal fail $\{ \}$

F(-1) timedout fail $\{ 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 \}$

F(-2) exception fail $\{ \}$

Sympy

A grade $\{ \}$

B grade $\{ 1, 2, 3, 4 \}$

C grade $\{ \}$

F normal fail $\{ 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 \}$

F(-1) timedout fail $\{ 9, 10, 11, 39, 51 \}$

F(-2) exception fail $\{ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	77	79	147	91	415	113	256
N.S.	1	1.00	0.64	0.65	1.21	0.75	3.43	0.93	2.12
time (sec)	N/A	0.137	0.295	1.326	0.193	0.268	0.374	0.439	14.083

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	57	57	123	77	301	77	212
N.S.	1	1.00	0.59	0.59	1.28	0.80	3.14	0.80	2.21
time (sec)	N/A	0.108	0.127	1.057	0.201	0.274	0.260	0.381	14.235

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	47	46	100	63	245	59	250
N.S.	1	1.00	0.61	0.60	1.30	0.82	3.18	0.77	3.25
time (sec)	N/A	0.080	0.111	0.789	0.199	0.267	0.179	0.367	14.057

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	44	77	46	133	59	125
N.S.	1	1.00	0.83	0.85	1.48	0.88	2.56	1.13	2.40
time (sec)	N/A	0.045	0.584	0.604	0.198	0.263	0.127	0.595	14.523

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	45	73	75	0	105	88
N.S.	1	1.00	0.97	0.71	1.16	1.19	0.00	1.67	1.40
time (sec)	N/A	0.070	0.136	0.463	0.201	0.271	0.000	0.331	12.674

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	97	61	69	99	0	129	110
N.S.	1	1.00	1.83	1.15	1.30	1.87	0.00	2.43	2.08
time (sec)	N/A	0.093	0.080	0.529	0.198	0.277	0.000	0.322	12.173

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	95	73	105	138	0	116	163
N.S.	1	1.00	1.48	1.14	1.64	2.16	0.00	1.81	2.55
time (sec)	N/A	0.087	0.627	0.776	0.200	0.283	0.000	0.318	12.172

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	172	95	120	137	0	139	132
N.S.	1	1.00	2.82	1.56	1.97	2.25	0.00	2.28	2.16
time (sec)	N/A	0.124	0.197	0.916	0.197	0.277	0.000	0.312	11.687

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	179	109	165	166	0	140	133
N.S.	1	1.00	2.08	1.27	1.92	1.93	0.00	1.63	1.55
time (sec)	N/A	0.126	0.225	1.070	0.201	0.260	0.000	0.403	11.530

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	204	123	187	201	0	174	244
N.S.	1	1.00	1.94	1.17	1.78	1.91	0.00	1.66	2.32
time (sec)	N/A	0.117	0.109	1.242	0.201	0.268	0.000	0.331	12.003

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	204	125	232	240	0	242	340
N.S.	1	1.00	1.57	0.96	1.78	1.85	0.00	1.86	2.62
time (sec)	N/A	0.148	0.125	1.327	0.204	0.267	0.000	0.383	12.226

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	165	69	78	0	155	0	99	0
N.S.	1	1.29	0.54	0.61	0.00	1.21	0.00	0.77	0.00
time (sec)	N/A	0.248	1.746	1.108	0.000	0.255	0.000	0.354	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	46	78	0	202	0	110	0
N.S.	1	1.00	0.67	1.13	0.00	2.93	0.00	1.59	0.00
time (sec)	N/A	0.229	1.700	0.513	0.000	0.278	0.000	0.344	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	74	124	0	328	0	141	0
N.S.	1	1.00	0.62	1.03	0.00	2.73	0.00	1.18	0.00
time (sec)	N/A	0.325	0.323	0.391	0.000	0.288	0.000	0.350	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	103	91	754	0	442	0	0	0
N.S.	1	1.00	0.88	7.32	0.00	4.29	0.00	0.00	0.00
time (sec)	N/A	0.317	2.595	3.211	0.000	0.410	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	37	309	41	0	0	52
N.S.	1	1.00	0.93	0.86	7.19	0.95	0.00	0.00	1.21
time (sec)	N/A	0.138	0.548	2.859	0.312	0.273	0.000	0.000	13.129

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	133	158	270	385	0	0	0
N.S.	1	1.00	1.17	1.39	2.37	3.38	0.00	0.00	0.00
time (sec)	N/A	0.339	0.219	3.342	0.320	0.360	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	132	130	0	391	0	0	0
N.S.	1	1.00	1.12	1.10	0.00	3.31	0.00	0.00	0.00
time (sec)	N/A	0.339	0.182	3.396	0.000	0.372	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	62	68	0	202	0	57	0
N.S.	1	1.00	1.35	1.48	0.00	4.39	0.00	1.24	0.00
time (sec)	N/A	0.125	0.072	1.565	0.000	0.380	0.000	0.336	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	64	100	59	0	0	1	0
N.S.	1	1.00	0.63	0.98	0.58	0.00	0.00	0.01	0.00
time (sec)	N/A	0.328	1.255	1.226	0.297	0.000	0.000	0.338	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	61	100	59	0	0	112	0
N.S.	1	1.00	0.61	1.00	0.59	0.00	0.00	1.12	0.00
time (sec)	N/A	0.332	0.774	1.243	0.296	0.000	0.000	0.338	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	63	91	190	193	0	0	0
N.S.	1	1.00	1.37	1.98	4.13	4.20	0.00	0.00	0.00
time (sec)	N/A	0.144	0.105	1.793	0.402	0.328	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	746	120	0	781	0	143	0
N.S.	1	1.00	7.10	1.14	0.00	7.44	0.00	1.36	0.00
time (sec)	N/A	0.211	6.013	0.582	0.000	0.581	0.000	0.343	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	654	206	0	1044	0	189	0
N.S.	1	1.00	3.96	1.25	0.00	6.33	0.00	1.15	0.00
time (sec)	N/A	0.331	3.366	0.590	0.000	1.513	0.000	0.396	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	662	935	0	3273	0	0	0
N.S.	1	1.00	4.44	6.28	0.00	21.97	0.00	0.00	0.00
time (sec)	N/A	0.349	3.395	3.638	0.000	1.475	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	83	83	436	505	0	1303	0	0	0
N.S.	1	1.00	5.25	6.08	0.00	15.70	0.00	0.00	0.00
time (sec)	N/A	0.148	2.711	3.166	0.000	0.887	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	166	166	280	589	0	3048	0	0	0
N.S.	1	1.00	1.69	3.55	0.00	18.36	0.00	0.00	0.00
time (sec)	N/A	0.358	5.903	3.142	0.000	1.330	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	168	168	289	580	0	3175	0	0	0
N.S.	1	1.00	1.72	3.45	0.00	18.90	0.00	0.00	0.00
time (sec)	N/A	0.360	8.216	3.490	0.000	1.576	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	452	593	0	0	0	0	0
N.S.	1	1.00	1.90	2.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	19.055	1.324	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	462	587	0	0	0	0	0
N.S.	1	1.00	1.88	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	5.986	1.252	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	267	267	13199	9956	0	0	0	0	0
N.S.	1	1.00	49.43	37.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	32.783	2.604	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	116	116	3415	3007	0	0	0	0	0
N.S.	1	1.00	29.44	25.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	28.982	2.158	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	252	252	4464	3057	0	0	0	0	0
N.S.	1	1.00	17.71	12.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	28.851	2.485	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	256	256	1667	3938	0	0	0	0	0
N.S.	1	1.00	6.51	15.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	19.746	2.612	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	123	123	567	5066	0	3539	0	0	0
N.S.	1	1.00	4.61	41.19	0.00	28.77	0.00	0.00	0.00
time (sec)	N/A	0.322	2.899	0.841	0.000	1.018	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	367	203	0	1044	0	0	0
N.S.	1	1.00	6.02	3.33	0.00	17.11	0.00	0.00	0.00
time (sec)	N/A	0.126	1.836	2.044	0.000	0.456	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	140	140	665	326	0	2791	0	0	0
N.S.	1	1.00	4.75	2.33	0.00	19.94	0.00	0.00	0.00
time (sec)	N/A	0.334	10.589	1.805	0.000	0.611	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	140	140	885	347	0	3005	0	0	0
N.S.	1	1.00	6.32	2.48	0.00	21.46	0.00	0.00	0.00
time (sec)	N/A	0.317	14.913	1.648	0.000	0.711	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	178	244	0	2837	0	292	23933
N.S.	1	1.00	0.98	1.35	0.00	15.67	0.00	1.61	132.23
time (sec)	N/A	0.345	1.321	2.033	0.000	65.065	0.000	0.441	27.413

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	179	190	0	0	0	0	0
N.S.	1	1.00	1.16	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	30.020	1.286	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	203	254	0	0	0	0	0
N.S.	1	1.00	1.39	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	29.589	1.148	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	254	254	23019	5478	0	0	0	0	0
N.S.	1	1.00	90.63	21.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	31.903	2.660	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	250	250	0	2884	0	0	0	0	0
N.S.	1	1.00	0.00	11.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.000	2.502	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	114	114	0	2583	0	0	0	0	0
N.S.	1	1.00	0.00	22.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.000	2.800	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	246	246	5612	3343	0	0	0	0	0
N.S.	1	1.00	22.81	13.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	31.093	2.860	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	254	254	23019	5489	0	0	0	0	0
N.S.	1	1.00	90.63	21.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	32.295	2.691	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	114	114	0	2590	0	0	0	0	0
N.S.	1	1.00	0.00	22.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.000	2.925	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	391	391	274	242134	0	0	0	0	0
N.S.	1	1.00	0.70	619.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.192	7.630	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	198	198	197	28726	0	0	0	0	0
N.S.	1	1.00	0.99	145.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.112	7.148	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	398	398	374	24290	0	0	0	0	0
N.S.	1	1.00	0.94	61.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	1.618	7.125	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [29] had the largest ratio of [.27269999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	13	4	1.00	32	0.125
2	A	11	4	1.00	32	0.125
3	A	10	5	1.00	30	0.167
4	A	4	4	1.00	24	0.167
5	A	6	5	1.00	30	0.167
6	A	8	7	1.00	32	0.219
7	A	7	5	1.00	32	0.156
8	A	6	6	1.00	32	0.188
9	A	11	5	1.00	32	0.156
10	A	11	4	1.00	32	0.125
11	A	13	4	1.00	32	0.125
12	A	5	5	1.29	34	0.147
13	A	5	5	1.00	34	0.147
14	A	8	7	1.00	34	0.206
15	A	6	6	1.00	40	0.150
16	A	3	3	1.00	40	0.075
17	A	6	6	1.00	40	0.150
18	A	6	6	1.00	40	0.150
19	A	2	2	1.00	36	0.056
20	A	6	6	1.00	36	0.167
21	A	6	6	1.00	36	0.167
22	A	3	3	1.00	36	0.083
23	A	5	4	1.00	33	0.121
24	A	8	6	1.00	33	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	39	0.103
26	A	2	2	1.00	39	0.051
27	A	5	4	1.00	39	0.103
28	A	5	4	1.00	39	0.103
29	A	9	9	1.00	33	0.273
30	A	9	9	1.00	33	0.273
31	A	3	3	1.00	39	0.077
32	A	1	1	1.00	39	0.026
33	A	3	3	1.00	39	0.077
34	A	3	3	1.00	39	0.077
35	A	5	5	1.00	35	0.143
36	A	2	2	1.00	35	0.057
37	A	5	5	1.00	35	0.143
38	A	5	5	1.00	35	0.143
39	A	8	5	1.00	33	0.152
40	A	5	3	1.00	33	0.091
41	A	5	3	1.00	33	0.091
42	A	3	3	1.00	39	0.077
43	A	3	3	1.00	39	0.077
44	A	1	1	1.00	39	0.026
45	A	3	3	1.00	39	0.077
46	A	3	3	1.00	39	0.077
47	A	1	1	1.00	39	0.026
48	A	3	3	1.00	35	0.086
49	A	1	1	1.00	35	0.029
50	A	3	3	1.00	35	0.086
51	A	4	4	1.00	38	0.105

CHAPTER 3

LISTING OF INTEGRALS

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Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{16}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos^5(e+fx)}{5f}-\frac{a^2c\cos(e+fx)\sin(e+fx)}{16f}-\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{24f}+\frac{a^2c\cos(e+fx)\sin^5(e+fx)}{6f}$$

```
[Out] 1/16*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f+1/5*a^2*c*cos(f*x+e)^5/f-1/16*a^2*c*c
os(f*x+e)*sin(f*x+e)/f-1/24*a^2*c*cos(f*x+e)*sin(f*x+e)^3/f+1/6*a^2*c*cos(f
*x+e)*sin(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 2713, 2715, 8}

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\cos^5(e+fx)}{5f}-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{a^2c\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{a^2c\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2cx$$

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/16 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]^5)/(5*f)
- (a^2*c*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a^2*c*Cos[e + f*x]*Sin[e + f*
x]^3)/(24*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*x]^5)/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(a^2 c \sin^3(e+fx) + a^2 c \sin^4(e+fx) - a^2 c \sin^5(e+fx) - a^2 c \sin^6(e+fx)\right) dx$$

$$= \left(a^2 c\right) \int \sin^3(e+fx)\,dx + \left(a^2 c\right) \int \sin^4(e+fx)\,dx - \left(a^2 c\right) \int \sin^5(e+fx)\,dx - \left(a^2 c\right) \int \sin^6(e+fx)\,dx$$

$$= -\frac{a^2 c \cos(e+fx) \sin^3(e+fx)}{4f} + \frac{a^2 c \cos(e+fx) \sin^5(e+fx)}{6f} + \frac{1}{4}\left(3a^2 c\right) \int \sin^2(e+fx)\,dx - \frac{1}{6}\left(5a^2 c\right) \int \sin^4(e+fx)\,dx - \frac{\left(a^2 c\right) \text{Subst}\left(\int \left(1-x^2\right) dx, x, \cos(e+fx)\right)}{f} + \frac{\left(a^2 c\right) \text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos^5(e+fx)}{5f} - \frac{3a^2c\cos(e+fx)\sin(e+fx)}{8f} - \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{24f} + \frac{a^2c\cos(e+fx)\sin^5(e+fx)}{6f} + \frac{1}{8}(3a^2c)\int 1\,dx - \frac{1}{8}(5a^2c)\int \sin^2(e+fx)\,dx$$

$$= \frac{3}{8}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos^5(e+fx)}{5f} - \frac{a^2c\cos(e+fx)\sin(e+fx)}{16f} - \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{24f} + \frac{a^2c\cos(e+fx)\sin^5(e+fx)}{6f} - \frac{1}{16}(5a^2c)\int 1\,dx$$

$$= \frac{1}{16}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos^5(e+fx)}{5f} - \frac{a^2c\cos(e+fx)\sin(e+fx)}{16f} - \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{24f} + \frac{a^2c\cos(e+fx)\sin^5(e+fx)}{6f}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2c(60e+60fx-120\cos(e+fx)-20\cos(3(e+fx))+12\cos(5(e+fx))-15\sin(2(e+fx))-15\sin(}{960f}$$

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
[Out] (a^2*c*(60*e + 60*f*x - 120*Cos[e + f*x] - 20*Cos[3*(e + f*x)] + 12*Cos[5*(
e + f*x)] - 15*Sin[2*(e + f*x)] - 15*Sin[4*(e + f*x)] + 5*Sin[6*(e + f*x)])
)/(960*f)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{a^2c(-60fx+120\cos(fx+e)-5\sin(6fx+6e)-12\cos(5fx+5e)+15\sin(4fx+4e)+20\cos(3fx+3e)+15\sin(2fx+2e)+128}{960f}$
risch	$\frac{a^2cx}{16}-\frac{a^2c\cos(fx+e)}{8f}+\frac{a^2c\sin(6fx+6e)}{192f}+\frac{a^2c\cos(5fx+5e)}{80f}-\frac{a^2c\sin(4fx+4e)}{64f}-\frac{a^2c\cos(3fx+3e)}{48f}-\frac{a^2c\sin}{6}$
derivativedivides	$\frac{-a^2c\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{a^2c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e}{5}}{f}$
default	$\frac{-a^2c\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{a^2c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e}{5}}{f}$
parts	$-\frac{a^2c(2+\sin^2(fx+e))\cos(fx+e)}{3f}+\frac{a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{a^2c\left(\frac{8}{3}+\sin^4(fx+e)+\right.}{}$
norman	$\frac{-\frac{4a^2c}{15f}+\frac{a^2cx}{16}-\frac{8a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{8a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f}-\frac{4a^2c\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{17a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}+}{}$

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/960*a^2*c*(-60*f*x+120*cos(f*x+e)-5*sin(6*f*x+6*e)-12*cos(5*f*x+5*e)+15*
sin(4*f*x+4*e)+20*cos(3*f*x+3*e)+15*sin(2*f*x+2*e)+128)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{48\,a^2c\cos\left(fx+e\right)^5-80\,a^2c\cos\left(fx+e\right)^3+15\,a^2cfx+5\left(8\,a^2c\cos\left(fx+e\right)^5-14\,a^2c\cos\left(fx+e\right)^3+3\,a^2\right.}{240\,f}$$

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(48*a^2*c*cos(f*x + e)^5 - 80*a^2*c*cos(f*x + e)^3 + 15*a^2*c*f*x + 5
*(8*a^2*c*cos(f*x + e)^5 - 14*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e))*
sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(110) = 220$.

Time = 0.37 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.43

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{5a^2cx\sin^6(e+fx)}{16} - \frac{15a^2cx\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{3a^2cx\sin^4(e+fx)}{8} - \frac{15a^2cx\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{3a^2cx\sin^2(e+fx)\cos \\ x(a\sin(e)+a)^2(-c\sin(e)+c)\sin^3(e) \end{cases}$$

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-5*a**2*c*x*sin(e + f*x)**6/16 - 15*a**2*c*x*sin(e + f*x)**4*cos
(e + f*x)**2/16 + 3*a**2*c*x*sin(e + f*x)**4/8 - 15*a**2*c*x*sin(e + f*x)**
2*cos(e + f*x)**4/16 + 3*a**2*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - 5*a**
2*c*x*cos(e + f*x)**6/16 + 3*a**2*c*x*cos(e + f*x)**4/8 + 11*a**2*c*sin(e +
 f*x)**5*cos(e + f*x)/(16*f) + a**2*c*sin(e + f*x)**4*cos(e + f*x)/f + 5*a*
*2*c*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*a**2*c*sin(e + f*x)**3*cos(e
 + f*x)/(8*f) + 4*a**2*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - a**2*c*sin
(e + f*x)**2*cos(e + f*x)/f + 5*a**2*c*sin(e + f*x)*cos(e + f*x)**5/(16*f)
- 3*a**2*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 8*a**2*c*cos(e + f*x)**5/(1
5*f) - 2*a**2*c*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*
sin(e) + c)*sin(e)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)a^2c + 320\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)a^2c -}{}$$

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c +
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c - 5*(4*sin(2*f*x + 2*e)^3 + 60*
f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*c + 30*(12*f*x +
 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c)/f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{16}a^2cx + \frac{a^2c\cos(5fx+5e)}{80f} - \frac{a^2c\cos(3fx+3e)}{48f} - \frac{a^2c\cos(fx+e)}{8f} + \frac{a^2c\sin(6fx+6e)}{192f} - \frac{a^2c\sin(4fx+4e)}{64f} - \frac{a^2c\sin(2fx+2e)}{64f}$$

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/16*a^2*c*x + 1/80*a^2*c*cos(5*f*x + 5*e)/f - 1/48*a^2*c*cos(3*f*x + 3*e)/
f - 1/8*a^2*c*cos(f*x + e)/f + 1/192*a^2*c*sin(6*f*x + 6*e)/f - 1/64*a^2*c*
sin(4*f*x + 4*e)/f - 1/64*a^2*c*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.12

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2\,c\,\left(15\,e-30\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-384\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-170\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+1140\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-640\,\tan\left(\frac{e}{2}+\right.\right.}{}$$

```
[In] int(sin(e + f*x)^3*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*(15*e - 30*tan(e/2 + (f*x)/2) - 384*tan(e/2 + (f*x)/2)^2 - 170*tan(e
/2 + (f*x)/2)^3 + 1140*tan(e/2 + (f*x)/2)^5 - 640*tan(e/2 + (f*x)/2)^6 - 11
40*tan(e/2 + (f*x)/2)^7 - 960*tan(e/2 + (f*x)/2)^8 + 170*tan(e/2 + (f*x)/2)
^9 + 30*tan(e/2 + (f*x)/2)^11 + 15*f*x + 90*tan(e/2 + (f*x)/2)^2*(e + f*x)
+ 225*tan(e/2 + (f*x)/2)^4*(e + f*x) + 300*tan(e/2 + (f*x)/2)^6*(e + f*x) +
 225*tan(e/2 + (f*x)/2)^8*(e + f*x) + 90*tan(e/2 + (f*x)/2)^10*(e + f*x) +
15*tan(e/2 + (f*x)/2)^12*(e + f*x) - 64))/(240*f*(tan(e/2 + (f*x)/2)^2 + 1)
^6)
```

3.2 $\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 96

$$\begin{aligned}\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx\\ &= \frac{1}{8}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos^5(e+fx)}{5f}\\ &\quad-\frac{a^2c\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}\end{aligned}$$

```
[Out] 1/8*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f+1/5*a^2*c*cos(f*x+e)^5/f-1/8*a^2*c*cos
(f*x+e)*sin(f*x+e)/f+1/4*a^2*c*cos(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 2715, 8, 2713}

$$\begin{aligned}\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx\\ &= \frac{a^2c\cos^5(e+fx)}{5f}-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin^3(e+fx)\cos(e+fx)}{4f}\\ &\quad-\frac{a^2c\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx\end{aligned}$$

```
[In] Int[Sin[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/8 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]^5)/(5*f) -
 (a^2*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*x]
^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2 c \sin^2(e+fx) + a^2 c \sin^3(e+fx) - a^2 c \sin^4(e+fx) - a^2 c \sin^5(e+fx)\right)\,dx \\
&= \left(a^2c\right)\int \sin^2(e+fx)\,dx + \left(a^2c\right)\int \sin^3(e+fx)\,dx \\
&\quad - \left(a^2c\right)\int \sin^4(e+fx)\,dx - \left(a^2c\right)\int \sin^5(e+fx)\,dx \\
&= -\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f} + \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{2}\left(a^2c\right)\int 1\,dx \\
&\quad - \frac{1}{4}\left(3a^2c\right)\int \sin^2(e+fx)\,dx - \frac{\left(a^2c\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(e+fx)\right)}{f} \\
&\quad + \frac{\left(a^2c\right)\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(e+fx)\right)}{f}
\end{aligned}$$

$$= \frac{1}{2}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos^5(e+fx)}{5f} - \frac{a^2c\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f} - \frac{1}{8}(3a^2c)\int 1\,dx$$

$$= \frac{1}{8}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos^5(e+fx)}{5f} - \frac{a^2c\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c(60e+60fx-60\cos(e+fx)-10\cos(3(e+fx))+6\cos(5(e+fx))-15\sin(4(e+fx)))}{480f}$$

```
[In] Integrate[Sin[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
[Out] (a^2*c*(60*e + 60*f*x - 60*Cos[e + f*x] - 10*Cos[3*(e + f*x)] + 6*Cos[5*(e
+ f*x)] - 15*Sin[4*(e + f*x)]))/(480*f)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

method	result
parallelrisch	$-\frac{a^2c(-60fx+60\cos(fx+e)-6\cos(5fx+5e)+15\sin(4fx+4e)+10\cos(3fx+3e)+64)}{480f}$
risch	$\frac{a^2cx}{8}-\frac{a^2c\cos(fx+e)}{8f}+\frac{a^2c\cos(5fx+5e)}{80f}-\frac{a^2c\sin(4fx+4e)}{32f}-\frac{a^2c\cos(3fx+3e)}{48f}$
derivativedivides	$\frac{\frac{a^2c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2c\left(2+\sin^2(fx+e\right.}{3}}{f}$
default	$\frac{\frac{a^2c\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}-a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2c\left(2+\sin^2(fx+e\right.}{3}}{f}$
parts	$\frac{a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{a^2c(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\right.}{f}$
norman	$\frac{-\frac{4a^2c}{15f}+\frac{a^2cx}{8}-\frac{4a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{4a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{3a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}-3}{}$

```
[In] int(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/480*a^2*c*(-60*f*x+60*cos(f*x+e)-6*cos(5*f*x+5*e)+15*sin(4*f*x+4*e)+10*c
os(3*f*x+3*e)+64)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{24\,a^2c\cos\left(fx+e\right)^5-40\,a^2c\cos\left(fx+e\right)^3+15\,a^2cfx-15\left(2\,a^2c\cos\left(fx+e\right)^3-a^2c\cos\left(fx+e\right)\right)\sin\left(fx\right.}{120\,f}$$

```
[In] integrate(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(24*a^2*c*cos(f*x + e)^5 - 40*a^2*c*cos(f*x + e)^3 + 15*a^2*c*f*x - 1
5*(2*a^2*c*cos(f*x + e)^3 - a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(87) = 174$.

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.14

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\begin{cases} -\frac{3a^2cx\sin^4(e+fx)}{8}-\frac{3a^2cx\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{a^2cx\sin^2(e+fx)}{2}-\frac{3a^2cx\cos^4(e+fx)}{8}+\frac{a^2cx\cos^2(e+fx)}{2}+\frac{a^2c\sin^4(e+f\ldots}{f} \\ x(a\sin(e)+a)^2(-c\sin(e)+c)\sin^2(e) \end{cases}$$

```
[In] integrate(sin(f*x+e)**2*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**2*c*x*sin(e + f*x)**4/8 - 3*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + a**2*c*x*sin(e + f*x)**2/2 - 3*a**2*c*x*cos(e + f*x)**4/8 +
a**2*c*x*cos(e + f*x)**2/2 + a**2*c*sin(e + f*x)**4*cos(e + f*x)/f + 5*a**2
*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 4*a**2*c*sin(e + f*x)**2*cos(e + f*
x)**3/(3*f) - a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*a**2*c*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*a**2*c*
cos(e + f*x)**5/(15*f) - 2*a**2*c*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a*s
in(e) + a)**2*(-c*sin(e) + c)*sin(e)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.28

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{32\left(3\cos(fx+e)^5-10\cos(fx+e)^3+15\cos(fx+e)\right)a^2c+160\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2c-\ldots}{480f}$$

```
[In] integrate(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c +
160*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c - 15*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*
a^2*c)/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{1}{8}\,a^2cx+\frac{a^2c\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{a^2c\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{a^2c\cos\left(fx+e\right)}{8\,f}-\frac{a^2c\sin\left(4\,fx+4\,e\right)}{32\,f}$$

```
[In] integrate(sin(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/8*a^2*c*x + 1/80*a^2*c*cos(5*f*x + 5*e)/f - 1/48*a^2*c*cos(3*f*x + 3*e)/f
 - 1/8*a^2*c*cos(f*x + e)/f - 1/32*a^2*c*sin(4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.21

$$\int \sin^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2\,c\,\left(15\,e-30\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-160\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+180\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+160\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-480\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}{}$$

```
[In] int(sin(e + f*x)^2*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*(15*e - 30*tan(e/2 + (f*x)/2) - 160*tan(e/2 + (f*x)/2)^2 + 180*tan(e
/2 + (f*x)/2)^3 + 160*tan(e/2 + (f*x)/2)^4 - 480*tan(e/2 + (f*x)/2)^6 - 180
*tan(e/2 + (f*x)/2)^7 + 30*tan(e/2 + (f*x)/2)^9 + 15*f*x + 75*tan(e/2 + (f*
x)/2)^2*(e + f*x) + 150*tan(e/2 + (f*x)/2)^4*(e + f*x) + 150*tan(e/2 + (f*x
)/2)^6*(e + f*x) + 75*tan(e/2 + (f*x)/2)^8*(e + f*x) + 15*tan(e/2 + (f*x)/2
)^10*(e + f*x) - 32))/(120*f*(tan(e/2 + (f*x)/2)^2 + 1)^5)
```

3.3 $\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 77

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{8}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}-\frac{a^2c\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}$$

```
[Out] 1/8*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f-1/8*a^2*c*cos(f*x+e)*sin(f*x+e)/f+1/4*
a^2*c*cos(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3045, 2718, 2715, 8, 2713}

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{a^2c\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx$$

```
[In] Int[Sin[e + f*x]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/8 - (a^2*c*Cos[e + f*x]^3)/(3*f) - (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(8*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2c\sin(e+fx)+a^2c\sin^2(e+fx)-a^2c\sin^3(e+fx)-a^2c\sin^4(e+fx)\right)\,dx\\
&= \left(a^2c\right)\int \sin(e+fx)\,dx+\left(a^2c\right)\int \sin^2(e+fx)\,dx\\
&\quad-\left(a^2c\right)\int \sin^3(e+fx)\,dx-\left(a^2c\right)\int \sin^4(e+fx)\,dx\\
&= -\frac{a^2c\cos(e+fx)}{f}-\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}+\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}\\
&\quad+\frac{1}{2}\left(a^2c\right)\int 1\,dx-\frac{1}{4}\left(3a^2c\right)\int \sin^2(e+fx)\,dx+\frac{\left(a^2c\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(e+fx)\right)}{f}\\
&= \frac{1}{2}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}-\frac{a^2c\cos(e+fx)\sin(e+fx)}{8f}\\
&\quad+\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}-\frac{1}{8}\left(3a^2c\right)\int 1\,dx\\
&= \frac{1}{8}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}-\frac{a^2c\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c\cos(e+fx)\sin^3(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2c(12e+12fx-24\cos(e+fx)-8\cos(3(e+fx))-3\sin(4(e+fx)))}{96f}$$

```
[In] Integrate[Sin[e + f*x]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
[Out] (a^2*c*(12*e + 12*f*x - 24*Cos[e + f*x] - 8*Cos[3*(e + f*x)] - 3*Sin[4*(e +
 f*x)]))/(96*f)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

method	result
parallelrisch	$-\frac{a^2c(-12fx+24\cos(fx+e)+3\sin(4fx+4e)+8\cos(3fx+3e)+32)}{96f}$
risch	$\frac{a^2cx}{8}-\frac{a^2c\cos(fx+e)}{4f}-\frac{a^2c\sin(4fx+4e)}{32f}-\frac{a^2c\cos(3fx+3e)}{12f}$
derivativedivides	$\frac{-a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{2}\right.}{f}$
default	$\frac{-a^2c\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{a^2c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{f}{2}\right.}{f}$
parts	$-\frac{a^2c\cos(fx+e)}{f}+\frac{a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2c(2+\sin^2(fx+e))\cos(fx+e)}{3f}-\frac{a^2c\left(-\frac{\left(\sin^3(fx+e)+\right.}{}\right.}{}$
norman	$\frac{-\frac{2a^2c}{3f}+\frac{a^2cx}{8}-\frac{2a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{7a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-}{\left(1+\mathrm{t}\right.}$

```
[In] int(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -1/96*a^2*c*(-12*f*x+24*cos(f*x+e)+3*sin(4*f*x+4*e)+8*cos(3*f*x+3*e)+32)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=-\frac{8\,a^2c\cos\left(fx+e\right)^3-3\,a^2cfx+3\,\left(2\,a^2c\cos\left(fx+e\right)^3-a^2c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

```
[In] integrate(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*a^2*c*cos(f*x + e)^3 - 3*a^2*c*f*x + 3*(2*a^2*c*cos(f*x + e)^3 - a
^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(70) = 140.

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.18

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\begin{cases}-\frac{3a^2cx\sin^4(e+fx)}{8}-\frac{3a^2cx\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{a^2cx\sin^2(e+fx)}{2}-\frac{3a^2cx\cos^4(e+fx)}{8}+\frac{a^2cx\cos^2(e+fx)}{2}+\frac{5a^2c\sin^3(e+fx}{8f}\\ x(a\sin(e)+a)^2\left(-c\sin(e)+c\right)\sin(e)\end{cases}$$

```
[In] integrate(sin(f*x+e)*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**2*c*x*sin(e + f*x)**4/8 - 3*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + a**2*c*x*sin(e + f*x)**2/2 - 3*a**2*c*x*cos(e + f*x)**4/8 +
a**2*c*x*cos(e + f*x)**2/2 + 5*a**2*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) +
a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*a**2*c*sin(e + f*x)*cos(e + f*x)*
*3/(8*f) - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*cos(e + f*x)**
3/(3*f) - a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e)
 + c)*sin(e), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$

$$-\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c+3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)a^2c-2\cdots}{96\,f}$$

```
[In] integrate(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c + 3*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*
e))*a^2*c + 96*a^2*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{8}\,a^2cx-\frac{a^2c\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^2c\cos\left(fx+e\right)}{4\,f}-\frac{a^2c\sin\left(4\,fx+4\,e\right)}{32\,f}$$

```
[In] integrate(sin(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/8*a^2*c*x - 1/12*a^2*c*cos(3*f*x + 3*e)/f - 1/4*a^2*c*cos(f*x + e)/f - 1/
32*a^2*c*sin(4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.25

$$\int \sin(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2\,c\,x}{8}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a^2\,c\,(3\,e+3\,f\,x)}{6}-\frac{a^2\,c\,(12\,e+12\,f\,x-16)}{24}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(\frac{a^2\,c\,(3\,e+3\,f\,x)}{6}-\frac{a^2\,c\,(12\,e+12\,f\,x-48)}{24}\right)+\mathrm{ta}\cdots}{\cdots}$$

```
[In] int(sin(e + f*x)*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*x)/8 - (tan(e/2 + (f*x)/2)^2*((a^2*c*(3*e + 3*f*x))/6 - (a^2*c*(12*e
 + 12*f*x - 16))/24) + tan(e/2 + (f*x)/2)^6*((a^2*c*(3*e + 3*f*x))/6 - (a^2
*c*(12*e + 12*f*x - 48))/24) + tan(e/2 + (f*x)/2)^4*((a^2*c*(3*e + 3*f*x))/
4 - (a^2*c*(18*e + 18*f*x - 48))/24) + (a^2*c*tan(e/2 + (f*x)/2))/4 - (7*a^
2*c*tan(e/2 + (f*x)/2)^3)/4 + (7*a^2*c*tan(e/2 + (f*x)/2)^5)/4 - (a^2*c*tan
(e/2 + (f*x)/2)^7)/4 + (a^2*c*(3*e + 3*f*x))/24 - (a^2*c*(3*e + 3*f*x - 16)
)/24)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)
```

3.4 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

Optimal result 59
Rubi [A] (verified) 59
Mathematica [A] (verified) 60
Maple [A] (verified) 61
Fricas [A] (verification not implemented) 61
Sympy [B] (verification not implemented) 62
Maxima [A] (verification not implemented) 62
Giac [A] (verification not implemented) 62
Mupad [B] (verification not implemented) 63

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx=\frac{1}{2}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f+1/2*a^2*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2815, 2748, 2715, 8}

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx=-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/2 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx \\
&= -\frac{a^2c\cos^3(e+fx)}{3f} + \left(a^2c\right)\int \cos^2(e+fx)\,dx \\
&= -\frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}\left(a^2c\right)\int 1\,dx \\
&= \frac{1}{2}a^2cx - \frac{a^2c\cos^3(e+fx)}{3f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{a^2c(3\cos(e+fx)+\cos(3(e+fx))-3(2fx+\sin(2(e+fx))))}{12f}$$

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
[Out] -1/12*(a^2*c*(3*Cos[e + f*x] + Cos[3*(e + f*x)] - 3*(2*f*x + Sin[2*(e + f*x
)])))/f
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result
parallelrisch	$-\frac{a^2 c(-6fx+3\cos(fx+e)+\cos(3fx+3e)-3\sin(2fx+2e)+4)}{12f}$
risch	$\frac{a^2 c x}{2}-\frac{a^2 c\cos(fx+e)}{4f}-\frac{a^2 c\cos(3fx+3e)}{12f}+\frac{a^2 c\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{\frac{a^2 c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a^2 c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2 c\cos(fx+e)+a^2 c(fx+e)}{f}$
default	$\frac{\frac{a^2 c\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-a^2 c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2 c\cos(fx+e)+a^2 c(fx+e)}{f}$
parts	$a^2 c x-\frac{a^2 c\cos(fx+e)}{f}-\frac{a^2 c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2 c(2+\sin^2(fx+e))\cos(fx+e)}{3f}$
norman	$\frac{\frac{a^2 c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a^2 c}{3f}+\frac{a^2 c x}{2}-\frac{2a^2 c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2 c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2 c x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a^2 c x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -1/12*a^2*c*(-6*f*x+3*cos(f*x+e)+cos(3*f*x+3*e)-3*sin(2*f*x+2*e)+4)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{2\,a^2 c\cos\left(fx+e\right)^3-3\,a^2 cfx-3\,a^2 c\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")
[Out] -1/6*(2*a^2*c*cos(f*x + e)^3 - 3*a^2*c*f*x - 3*a^2*c*cos(f*x + e)*sin(f*x +
 e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(46) = 92.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.56

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{a^2cx\sin^2(e+fx)}{2} - \frac{a^2cx\cos^2(e+fx)}{2} + a^2cx + \frac{a^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c\cos^3(e+fx)}{3f} - \frac{a^2c \\ x(a\sin(e)+a)^2(-c\sin(e)+c) \end{cases}$$

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a**2*c*x*sin(e + f*x)**2/2 - a**2*c*x*cos(e + f*x)**2/2 + a**2*
c*x + a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + a**2*c*sin(e + f*x)*cos(e + f
*x)/(2*f) + 2*a**2*c*cos(e + f*x)**3/(3*f) - a**2*c*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{4\left(\cos(fx+e)^3-3\cos(fx+e)\right)a^2c+3\left(2fx+2e-\sin(2fx+2e)\right)a^2c-12(fx+e)a^2c+12a^2c\cos}{12f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c + 3*(2*f*x + 2*e - sin(2*f
*x + 2*e))*a^2*c - 12*(f*x + e)*a^2*c + 12*a^2*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{2}a^2cx - \frac{a^2c\cos(3fx+3e)}{12f} - \frac{a^2c\cos(fx+e)}{4f} + \frac{a^2c\sin(2fx+2e)}{4f}$$

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c*x - 1/12*a^2*c*cos(3*f*x + 3*e)/f - 1/4*a^2*c*cos(f*x + e)/f + 1/
4*a^2*c*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2\,c\,x}{2} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{3\,a^2\,c\,(e+f\,x)}{2}-\frac{a^2\,c\,(9\,e+9\,f\,x-12)}{6}\right)-a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a^2\,c\,(e+f\,x)}{2}+a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{a^2\,c}{}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((3*a^2*c*(e + f*x))/2 - (a^2*c*(9*e +
9*f*x - 12))/6) - a^2*c*tan(e/2 + (f*x)/2) + (a^2*c*(e + f*x))/2 + a^2*c*ta
n(e/2 + (f*x)/2)^5 - (a^2*c*(3*e + 3*f*x - 4))/6)/(f*(tan(e/2 + (f*x)/2)^2
+ 1)^3)
```

3.5 $\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Rubi [A] (verified) 64
Mathematica [A] (verified) 66
Maple [A] (verified) 66
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Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{1}{2}a^2cx - \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{f} + \frac{a^2c\cos(e+fx)}{f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*x-a^2*c*arctanh(cos(f*x+e))/f+a^2*c*cos(f*x+e)/f+1/2*a^2*c*cos(f*
x+e)*sin(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3045, 3855, 2718, 2715, 8}

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{f} + \frac{a^2c\cos(e+fx)}{f} + \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{1}{2}a^2cx$$

```
[In] Int[Csc[e + f*x]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/2 - (a^2*c*ArcTanh[Cos[e + f*x]])/f + (a^2*c*Cos[e + f*x])/f + (a
^2*c*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2c + a^2c\csc(e+fx) - a^2c\sin(e+fx) - a^2c\sin^2(e+fx)\right)\,dx\\
&= a^2cx + \left(a^2c\right)\int \csc(e+fx)\,dx - \left(a^2c\right)\int \sin(e+fx)\,dx - \left(a^2c\right)\int \sin^2(e+fx)\,dx\\
&= a^2cx - \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{f} + \frac{a^2c\cos(e+fx)}{f}\\
&\quad + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f} - \frac{1}{2}\left(a^2c\right)\int 1\,dx\\
&= \frac{1}{2}a^2cx - \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{f} + \frac{a^2c\cos(e+fx)}{f} + \frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\left(-2e+2fx+4\cos(e+fx)-4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sin(2(e+fx))\right)}{4f}$$

```
[In] Integrate[Csc[e + f*x]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
[Out] (a^2*c*(-2*e + 2*f*x + 4*Cos[e + f*x] - 4*Log[Cos[(e + f*x)/2]] + 4*Log[Sin
[(e + f*x)/2]] + Sin[2*(e + f*x)]))/(4*f)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{a^2c\left(2fx+4\cos(fx+e)+4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\sin(2fx+2e)+4\right)}{4f}$
derivativedivides	$\frac{-a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2c\cos(fx+e)+a^2c(fx+e)+a^2c\ln(\csc(fx+e)-\cot(fx+e))}{f}$
default	$\frac{-a^2c\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2c\cos(fx+e)+a^2c(fx+e)+a^2c\ln(\csc(fx+e)-\cot(fx+e))}{f}$
risch	$\frac{a^2cx}{2}+\frac{a^2c\,e^{i(fx+e)}}{2f}+\frac{a^2c\,e^{-i(fx+e)}}{2f}-\frac{a^2c\ln\left(e^{i(fx+e)}+1\right)}{f}+\frac{a^2c\ln\left(e^{i(fx+e)}-1\right)}{f}+\frac{a^2c\sin(2fx+2e)}{4f}$
norman	$\frac{\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2cx}{2}-\frac{2a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{3a^2cx\left(\tan^2\right.}{2}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

```
[In] int(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*c*(2*f*x+4*cos(f*x+e)+4*ln(tan(1/2*f*x+1/2*e))+sin(2*f*x+2*e)+4)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2cfx+a^2c\cos(fx+e)\sin(fx+e)+2\,a^2c\cos(fx+e)-a^2c\log\left(\frac{1}{2}\,\cos(fx+e)+\frac{1}{2}\right)+a^2c\log\left(-\frac{1}{2}\,\cos\right.}{2\,f}$$

```
[In] integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*c*f*x + a^2*c*cos(f*x + e)*sin(f*x + e) + 2*a^2*c*cos(f*x + e) - a
^2*c*log(1/2*cos(f*x + e) + 1/2) + a^2*c*log(-1/2*cos(f*x + e) + 1/2))/f
```

Sympy [F]

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= -a^2c\left(\int(-\sin(e+fx)\csc(e+fx))\,dx + \int\sin^2(e+fx)\csc(e+fx)\,dx + \int\sin^3(e+fx)\csc(e+fx)\,dx + \int(-\csc(e+fx))\,dx\right)$$

```
[In] integrate(csc(f*x+e)*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sin(e + f*x)*csc(e + f*x), x) + Integral(sin(e + f*x)**2
*csc(e + f*x), x) + Integral(sin(e + f*x)**3*csc(e + f*x), x) + Integral(-c
sc(e + f*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{(2\,fx+2\,e-\sin(2\,fx+2\,e))a^2c-4\,(fx+e)a^2c-4\,a^2c\cos(fx+e)+4\,a^2c\log(\cot(fx+e)+\csc(f}{4\,f}$$

```
[In] integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c - 4*(f*x + e)*a^2*c - 4*a^2*c*
cos(f*x + e) + 4*a^2*c*log(cot(f*x + e) + csc(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{(fx+e)a^2c+2\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-\frac{2\left(a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,a^2c\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2}}{2\,f}$$

```
[In] integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((f*x + e)*a^2*c + 2*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 2*(a^2*c*ta
n(1/2*f*x + 1/2*e)^3 - 2*a^2*c*tan(1/2*f*x + 1/2*e)^2 - a^2*c*tan(1/2*f*x +
 1/2*e) - 2*a^2*c)/(tan(1/2*f*x + 1/2*e)^2 + 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \csc(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2\,c\left(\cos\left(e+f\,x\right)+\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)+\frac{\sin(2\,e+2\,f\,x)}{4}+\mathrm{atan}\left(\frac{\sqrt{5}\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{5\,\cos\left(\frac{e}{2}+\mathrm{atan}\left(\frac{1}{2}\right)+\frac{f\,x}{2}\right)}\right)\right)}{f}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x),x)
```

```
[Out] (a^2*c*(cos(e + f*x) + log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + sin(2*e
 + 2*f*x)/4 + atan((5^(1/2)*(cos(e/2 + (f*x)/2) + 2*sin(e/2 + (f*x)/2)))/(5
*cos(e/2 + atan(1/2) + (f*x)/2)))))/f
```

3.6 $\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Rubi [A] (verified) 69
Mathematica [A] (verified) 71
Maple [A] (verified) 71
Fricas [A] (verification not implemented) 72
Sympy [F] 72
Maxima [A] (verification not implemented) 73
Giac [B] (verification not implemented) 73
Mupad [B] (verification not implemented) 74

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -a^2cx-\frac{a^2c\,\text{arctanh}(\cos(e+fx))}{f}+\frac{a^2c\cos(e+fx)}{f}-\frac{a^2c\cot(e+fx)}{f}$$

```
[Out] -a^2*c*x-a^2*c*arctanh(cos(f*x+e))/f+a^2*c*cos(f*x+e)/f-a^2*c*cot(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3029, 2789, 2672, 327, 212, 3554, 8}

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{a^2c\,\text{arctanh}(\cos(e+fx))}{f}+\frac{a^2c\cos(e+fx)}{f}-\frac{a^2c\cot(e+fx)}{f}+a^2(-c)x$$

```
[In] Int[Csc[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(a^2*c*x) - (a^2*c*ArcTanh[Cos[e + f*x]])/f + (a^2*c*Cos[e + f*x])/f - (a^
2*c*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = (ac)\int \cot^2(e+fx)(a+a\sin(e+fx))\,dx$$

$$= (ac)\int \left(a\cos(e+fx)\cot(e+fx)+a\cot^2(e+fx)\right)\,dx$$

$$= \left(a^2c\right)\int \cos(e+fx)\cot(e+fx)\,dx+\left(a^2c\right)\int \cot^2(e+fx)\,dx$$

$$= -\frac{a^2c\cot(e+fx)}{f}-\left(a^2c\right)\int 1\,dx-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -a^2cx+\frac{a^2c\cos(e+fx)}{f}-\frac{a^2c\cot(e+fx)}{f}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -a^2cx-\frac{a^2c\,\text{arctanh}(\cos(e+fx))}{f}+\frac{a^2c\cos(e+fx)}{f}-\frac{a^2c\cot(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= -a^2cx+\frac{a^2c\cos(e)\cos(fx)}{f}-\frac{a^2c\cot(e+fx)}{f}-\frac{a^2c\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{a^2c\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a^2c\sin(e)\sin(fx)}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(a^2*c*x) + (a^2*c*Cos[e]*Cos[f*x])/f - (a^2*c*Cot[e + f*x])/f - (a^2*c*Lo
g[Cos[e/2 + (f*x)/2]])/f + (a^2*c*Log[Sin[e/2 + (f*x)/2]])/f - (a^2*c*Sin[e
]*Sin[f*x])/f
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^2c\cos(fx+e)-a^2c(fx+e)+a^2c\ln(\csc(fx+e)-\cot(fx+e))-a^2c\cot(fx+e)}{f}$
default	$\frac{a^2c\cos(fx+e)-a^2c(fx+e)+a^2c\ln(\csc(fx+e)-\cot(fx+e))-a^2c\cot(fx+e)}{f}$
parallelrisch	$\frac{a^2c\left(-2fx-2+2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2\cos(fx+e)+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$
risch	$-a^2cx+\frac{a^2c\,e^{i(fx+e)}}{2f}+\frac{a^2c\,e^{-i(fx+e)}}{2f}-\frac{2ia^2c}{f\left(e^{2i(fx+e)}-1\right)}-\frac{a^2c\ln\left(e^{i(fx+e)}+1\right)}{f}+\frac{a^2c\ln\left(e^{i(fx+e)}-1\right)}{f}$
norman	$\frac{\frac{a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c}{2f}-\frac{2a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2a^2c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\frac{a^2c\left(\tan^8\right.}{2}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(1+\tan^2\left(\frac{fx}{2}\right.\right.}$

```
[In] int(csc(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^2*c*cos(f*x+e)-a^2*c*(f*x+e)+a^2*c*ln(csc(f*x+e)-cot(f*x+e))-a^2*c*c
ot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{a^2c\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-a^2c\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2\,a^2c\cos\left(fx+e\right.}{2\,f\sin\left(fx+e\right)}$$

```
[In] integrate(csc(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(a^2*c*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - a^2*c*log(-1/2*cos(f
*x + e) + 1/2)*sin(f*x + e) + 2*a^2*c*cos(f*x + e) + 2*(a^2*c*f*x - a^2*c*c
os(f*x + e))*sin(f*x + e))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= -a^2c\left(\int\left(-\sin\left(e+fx\right)\csc^2\left(e+fx\right)\right)\,dx+\int\sin^2\left(e+fx\right)\csc^2\left(e+fx\right)dx\right.$$
$$\left.+\int\sin^3\left(e+fx\right)\csc^2\left(e+fx\right)dx+\int\left(-\csc^2\left(e+fx\right)\right)\,dx\right)$$

```
[In] integrate(csc(f*x+e)**2*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sin(e + f*x)*csc(e + f*x)**2, x) + Integral(sin(e + f*x)
**2*csc(e + f*x)**2, x) + Integral(sin(e + f*x)**3*csc(e + f*x)**2, x) + In
tegral(-csc(e + f*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{2\,(fx+e)a^2c+a^2c(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right))-2\,a^2c\cos\left(fx+e\right)+\frac{2\,a^2c}{\tan(fx+e)}}{2\,f}$$

```
[In] integrate(csc(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(f*x + e)*a^2*c + a^2*c*(log(cos(f*x + e) + 1) - log(cos(f*x + e) -
 1)) - 2*a^2*c*cos(f*x + e) + 2*a^2*c/tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.43

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{6\,(fx+e)a^2c-6\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{2\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\mathrm{ta}}}{6\,f}$$

```
[In] integrate(csc(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(f*x + e)*a^2*c - 6*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 3*a^2*c*
tan(1/2*f*x + 1/2*e) + (2*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*
x + 1/2*e)^2 - 10*a^2*c*tan(1/2*f*x + 1/2*e) + 3*a^2*c)/(tan(1/2*f*x + 1/2*
e)^3 + tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \csc^2(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2\,c\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}\right)+\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{f}$$
$$-\frac{a^2\,c\,\left(\cos\left(e+f\,x\right)-\frac{\sin(2\,e+2\,f\,x)}{2}\right)}{f\,\sin\left(e+f\,x\right)}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^2,x)
```

```
[Out] (a^2*c*(2*atan((2^(1/2)*(cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)))/(2*cos(e
/2 - pi/4 + (f*x)/2))) + log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/f - (
a^2*c*(cos(e + f*x) - sin(2*e + 2*f*x)/2))/(f*sin(e + f*x))
```

3.7 $\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 64

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -a^2cx+\frac{a^2c\,\text{arctanh}(\cos(e+fx))}{2f}-\frac{a^2c\cot(e+fx)}{f}-\frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -a^2*c*x+1/2*a^2*c*arctanh(cos(f*x+e))/f-a^2*c*cot(f*x+e)/f-1/2*a^2*c*cot(f
*x+e)*csc(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3045, 3855, 3852, 8, 3853}

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{2f}-\frac{a^2c\cot(e+fx)}{f}-\frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}+a^2(-c)x$$

```
[In] Int[Csc[e + f*x]^3*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(a^2*c*x) + (a^2*c*ArcTanh[Cos[e + f*x]])/(2*f) - (a^2*c*Cot[e + f*x])/f -
 (a^2*c*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-a^2c - a^2c\csc(e+fx) + a^2c\csc^2(e+fx) + a^2c\csc^3(e+fx)\right)\,dx \\
&= -a^2cx - \left(a^2c\right)\int \csc(e+fx)\,dx + \left(a^2c\right)\int \csc^2(e+fx)\,dx + \left(a^2c\right)\int \csc^3(e+fx)\,dx \\
&= -a^2cx + \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{f} - \frac{a^2c\cot(e+fx)\csc(e+fx)}{2f} \\
&\quad + \frac{1}{2}\left(a^2c\right)\int \csc(e+fx)\,dx - \frac{\left(a^2c\right)\operatorname{Subst}\left(\int 1\,dx, x, \cot(e+fx)\right)}{f} \\
&= -a^2cx + \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{2f} - \frac{a^2c\cot(e+fx)}{f} - \frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{a^2c\left(8e+8fx+4\cot\left(\frac{1}{2}(e+fx)\right)+\csc^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}{8f}$$

```
[In] Integrate[Csc[e + f*x]^3*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/8*(a^2*c*(8*e + 8*f*x + 4*Cot[(e + f*x)/2] + Csc[(e + f*x)/2]^2 - 4*Log[
Cos[(e + f*x)/2]] + 4*Log[Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2 - 4*Tan[(e
 + f*x)/2]))/f
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-\left(\cot^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-8fx+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-4\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}$
derivativedivides	$\frac{-a^2c(fx+e)-a^2c\ln(\csc(fx+e)-\cot(fx+e))-a^2c\cot(fx+e)+a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{-a^2c(fx+e)-a^2c\ln(\csc(fx+e)-\cot(fx+e))-a^2c\cot(fx+e)+a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$-a^2cx+\frac{a^2c\left(e^{3i(fx+e)}+e^{i(fx+e)}-2ie^{2i(fx+e)}+2i\right)}{f\left(e^{2i(fx+e)}-1\right)^2}+\frac{a^2c\ln\left(e^{i(fx+e)}+1\right)}{2f}-\frac{a^2c\ln\left(e^{i(fx+e)}-1\right)}{2f}$
norman	$\frac{\frac{a^2c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2c}{8f}-\frac{3a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{7a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{11a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}-\frac{a^2c\left(\tan^3\right.}{}}{\tan}$

```
[In] int(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8*a^2*c*(tan(1/2*f*x+1/2*e)^2-cot(1/2*f*x+1/2*e)^2-8*f*x+4*tan(1/2*f*x+1/
2*e)-4*ln(tan(1/2*f*x+1/2*e))-4*cot(1/2*f*x+1/2*e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{4\,a^2cfx\cos\left(fx+e\right)^2-4\,a^2cfx-4\,a^2c\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^2c\cos\left(fx+e\right)-\left(a^2c\cos\left(fx+e\right)\right.}{4\left(f\cos\left(fx+e\right)^2-\right.}$$

```
[In] integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(4*a^2*c*f*x*cos(f*x + e)^2 - 4*a^2*c*f*x - 4*a^2*c*cos(f*x + e)*sin(f
*x + e) - 2*a^2*c*cos(f*x + e) - (a^2*c*cos(f*x + e)^2 - a^2*c)*log(1/2*cos
(f*x + e) + 1/2) + (a^2*c*cos(f*x + e)^2 - a^2*c)*log(-1/2*cos(f*x + e) + 1
/2))/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\begin{aligned}\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx &= -a^2c\left(\int\left(-\sin\left(e+fx\right)\csc^3\left(e+fx\right)\right)dx+\int\sin^2\left(e+fx\right)\csc^3\left(e+fx\right)dx\right.\\&\quad\left.+\int\sin^3\left(e+fx\right)\csc^3\left(e+fx\right)dx+\int\left(-\csc^3\left(e+fx\right)\right)dx\right)\end{aligned}$$

```
[In] integrate(csc(f*x+e)**3*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sin(e + f*x)*csc(e + f*x)**3, x) + Integral(sin(e + f*x)
**2*csc(e + f*x)**3, x) + Integral(sin(e + f*x)**3*csc(e + f*x)**3, x) + In
tegral(-csc(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.64

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -\frac{4\left(fx+e\right)a^2c-a^2c\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-2\,a^2c(\log\left(\cos\left(fx\right.\right.}{4\,f}$$

```
[In] integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*(f*x + e)*a^2*c - a^2*c*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(
cos(f*x + e) + 1) + log(cos(f*x + e) - 1)) - 2*a^2*c*(log(cos(f*x + e) + 1)
 - log(cos(f*x + e) - 1)) + 4*a^2*c/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.81

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-8\,(fx+e)a^2c-4\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)+4\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{6\,a^2c\tan(\frac{1}{2}}{}}{8\,f}$$

```
[In] integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/8*(a^2*c*tan(1/2*f*x + 1/2*e)^2 - 8*(f*x + e)*a^2*c - 4*a^2*c*log(abs(tan
(1/2*f*x + 1/2*e))) + 4*a^2*c*tan(1/2*f*x + 1/2*e) + (6*a^2*c*tan(1/2*f*x +
 1/2*e)^2 - 4*a^2*c*tan(1/2*f*x + 1/2*e) - a^2*c)/tan(1/2*f*x + 1/2*e)^2)/f
```

Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.55

$$\int \csc^3(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}-\frac{a^2\,c\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{2\,f}-\frac{2\,a^2\,c\,\mathrm{atan}\left(\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}$$
$$-\frac{a^2\,c\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,f}-\frac{a^2\,c\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^3,x)
```

```
[Out] (a^2*c*tan(e/2 + (f*x)/2))/(2*f) - (a^2*c*log(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2)))/(2*f) - (2*a^2*c*atan((2*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)
)/(cos(e/2 + (f*x)/2) - 2*sin(e/2 + (f*x)/2))))/f - (a^2*c*cot(e/2 + (f*x)/
2))/(2*f) - (a^2*c*cot(e/2 + (f*x)/2)^2)/(8*f) + (a^2*c*tan(e/2 + (f*x)/2)^
2)/(8*f)
```

3.8 $\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

Optimal result 80
Rubi [A] (verified) 80
Mathematica [B] (verified) 82
Maple [A] (verified) 82
Fricas [B] (verification not implemented) 83
Sympy [F] 84
Maxima [B] (verification not implemented) 84
Giac [B] (verification not implemented) 84
Mupad [B] (verification not implemented) 85

Optimal result

Integrand size = 32, antiderivative size = 61

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{2f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*arctanh(cos(f*x+e))/f-1/3*a^2*c*cot(f*x+e)^3/f-1/2*a^2*c*cot(f*x+
e)*csc(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3029, 2785, 2687, 30, 2691, 3855}

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{2f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}$$

```
[In] Int[Csc[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Cos[e + f*x]])/(2*f) - (a^2*c*Cot[e + f*x]^3)/(3*f) - (a^2*c
*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^2c^2\right)\int \frac{\cot^4(e+fx)}{c-c\sin(e+fx)}\,dx\\ &= \left(a^2c\right)\int \cot^2(e+fx)\csc(e+fx)\,dx+\left(a^2c\right)\int \cot^2(e+fx)\csc^2(e+fx)\,dx\end{aligned}$$

$$= -\frac{a^2c\cot(e+fx)\csc(e+fx)}{2f} - \frac{1}{2}(a^2c)\int\csc(e+fx)\,dx + \frac{(a^2c)\,\mathrm{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{f}$$

$$= \frac{a^2c\,\mathrm{arctanh}(\cos(e+fx))}{2f} - \frac{a^2c\cot^3(e+fx)}{3f} - \frac{a^2c\cot(e+fx)\csc(e+fx)}{2f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(61) = 122$.

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int\csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= a^2c\left(\frac{\cot\left(\frac{1}{2}(e+fx)\right)}{6f} - \frac{\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{24f} + \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{\tan\left(\frac{1}{2}(e+fx)\right)}{6f} + \frac{\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{24f}\right)$$

```
[In] Integrate[Csc[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] a^2*c*(Cot[(e + f*x)/2]/(6*f) - Csc[(e + f*x)/2]^2/(8*f) - (Cot[(e + f*x)/2
]*Csc[(e + f*x)/2]^2)/(24*f) + Log[Cos[(e + f*x)/2]]/(2*f) - Log[Sin[(e + f
*x)/2]]/(2*f) + Sec[(e + f*x)/2]^2/(8*f) - Tan[(e + f*x)/2]/(6*f) + (Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/(24*f))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

method	result
parallelrisch	$\frac{a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)-\left(\cot^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3\left(\cot^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3\,\mathrm{c}}{24f}$
derivativedivides	$\frac{-a^2c\ln(\csc(fx+e)-\cot(fx+e))+a^2c\cot(fx+e)+a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\frac{(\mathrm{cs}}{}\right.}{f}$
default	$\frac{-a^2c\ln(\csc(fx+e)-\cot(fx+e))+a^2c\cot(fx+e)+a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\frac{(\mathrm{cs}}{}\right.}{f}$
risch	$\frac{a^2c\left(6i\,\mathrm{e}^{4i(fx+e)}+3\,\mathrm{e}^{5i(fx+e)}+2i-3\,\mathrm{e}^{i(fx+e)}\right)}{3f\left(\mathrm{e}^{2i(fx+e)}-1\right)^3}+\frac{a^2c\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{2f}-\frac{a^2c\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{2f}$
norman	$\frac{-\frac{a^2c}{24f}-\frac{3a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{7a^2c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{11a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{a^2c\left(\tan\right.}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}$

```
[In] int(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*a^2*c*(tan(1/2*f*x+1/2*e)^3-cot(1/2*f*x+1/2*e)^3+3*tan(1/2*f*x+1/2*e)^
2-3*cot(1/2*f*x+1/2*e)^2-3*tan(1/2*f*x+1/2*e)-12*ln(tan(1/2*f*x+1/2*e))+3*c
ot(1/2*f*x+1/2*e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.25

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{4\,a^2c\cos\left(fx+e\right)^3+6\,a^2c\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,\left(a^2c\cos\left(fx+e\right)^2-a^2c\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{12\,\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(f\right.}$$

```
[In] integrate(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/12*(4*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)*sin(f*x + e) + 3*(a^2*c
*cos(f*x + e)^2 - a^2*c)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(a^2*
c*cos(f*x + e)^2 - a^2*c)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e))/((f*co
s(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = -a^2c\left(\int\left(-\sin\left(e+fx\right)\csc^4\left(e+fx\right)\right)\,dx+\int\sin^2\left(e+fx\right)\csc^4\left(e+fx\right)dx + \int\sin^3\left(e+fx\right)\csc^4\left(e+fx\right)dx+\int\left(-\csc^4\left(e+fx\right)\right)\,dx\right)$$

```
[In] integrate(csc(f*x+e)**4*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
[Out] -a**2*c*(Integral(-sin(e + f*x)*csc(e + f*x)**4, x) + Integral(sin(e + f*x)
**2*csc(e + f*x)**4, x) + Integral(sin(e + f*x)**3*csc(e + f*x)**4, x) + In
tegral(-csc(e + f*x)**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(55) = 110.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{3\,a^2c\left(\frac{2\,\cos(fx+e)}{\cos(fx+e)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)+6\,a^2c(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\mathrm{c}\right.}{12\,f}$$

```
[In] integrate(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
[Out] 1/12*(3*a^2*c*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1)
+ log(cos(f*x + e) - 1)) + 6*a^2*c*(log(cos(f*x + e) + 1) - log(cos(f*x + e
) - 1)) + 12*a^2*c/tan(f*x + e) - 4*(3*tan(f*x + e)^2 + 1)*a^2*c/tan(f*x +
e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)-3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+}{24\,f}$$

[In] integrate(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] 1/24*(a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 3*a^2*c*tan(1/2*f*x + 1/2*e) + (22*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*c*tan(1/2*f*x + 1/2*e) - a^2*c)/tan(1/2*f*x + 1/2*e)^3)/f

Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int \csc^4(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}-\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8\,f} - \frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(-c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{c\,a^2}{3}\right)}{8\,f} + \frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24\,f}-\frac{a^2\,c\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^4,x)

[Out] (a^2*c*tan(e/2 + (f*x)/2)^2)/(8*f) - (a^2*c*tan(e/2 + (f*x)/2))/(8*f) - (cot(e/2 + (f*x)/2)^3*((a^2*c)/3 + a^2*c*tan(e/2 + (f*x)/2) - a^2*c*tan(e/2 + (f*x)/2)^2))/(8*f) + (a^2*c*tan(e/2 + (f*x)/2)^3)/(24*f) - (a^2*c*log(tan(e/2 + (f*x)/2)))/(2*f)

3.9 $\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

Optimal result 86
Rubi [A] (verified) 86
Mathematica [B] (verified) 88
Maple [A] (verified) 89
Fricas [B] (verification not implemented) 89
Sympy [F(-1)] 90
Maxima [B] (verification not implemented) 90
Giac [A] (verification not implemented) 90
Mupad [B] (verification not implemented) 91

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{8f}-\frac{a^2c\cot^3(e+fx)}{3f}+\frac{a^2c\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f}$$

```
[Out] 1/8*a^2*c*arctanh(cos(f*x+e))/f-1/3*a^2*c*cot(f*x+e)^3/f+1/8*a^2*c*cot(f*x+
e)*csc(f*x+e)/f-1/4*a^2*c*cot(f*x+e)*csc(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3045, 3852, 8, 3853, 3855}

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\cos(e+fx))}{8f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f}+\frac{a^2c\cot(e+fx)\csc(e+fx)}{8f}$$

```
[In] Int[Csc[e + f*x]^5*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Cos[e + f*x]])/(8*f) - (a^2*c*Cot[e + f*x]^3)/(3*f) + (a^2*c
*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (a^2*c*Cot[e + f*x]*Csc[e + f*x]^3)/(4*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(-a^2 c \csc^2(e+fx) - a^2 c \csc^3(e+fx) + a^2 c \csc^4(e+fx) + a^2 c \csc^5(e+fx)\right) \, dx$$

$$= -\left(\left(a^2 c\right) \int \csc^2(e+fx) \, dx\right) - \left(a^2 c\right) \int \csc^3(e+fx) \, dx + \left(a^2 c\right) \int \csc^4(e+fx) \, dx + \left(a^2 c\right) \int \csc^5(e+fx) \, dx$$

$$= \frac{a^2c\cot(e+fx)\csc(e+fx)}{2f} - \frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{1}{2}(a^2c)\int\csc(e+fx)\,dx + \frac{1}{4}(3a^2c)\int\csc^3(e+fx)\,dx + \frac{(a^2c)\,\mathrm{Subst}(\int 1\,dx,x,\cot(e+fx))}{f} - \frac{(a^2c)\,\mathrm{Subst}(\int(1+x^2)\,dx,x,\cot(e+fx))}{f}$$

$$= \frac{a^2c\,\mathrm{arctanh}(\cos(e+fx))}{2f} - \frac{a^2c\cot^3(e+fx)}{3f} + \frac{a^2c\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{1}{8}(3a^2c)\int\csc(e+fx)\,dx$$

$$= \frac{a^2c\,\mathrm{arctanh}(\cos(e+fx))}{8f} - \frac{a^2c\cot^3(e+fx)}{3f} + \frac{a^2c\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.08

$$\int\csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\cot(e+fx)}{3f} + \frac{a^2c\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} - \frac{a^2c\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{a^2c\cot(e+fx)\csc^2(e+fx)}{3f} + \frac{a^2c\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{a^2c\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{a^2c\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a^2c\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f}$$

```
[In] Integrate[Csc[e + f*x]^5*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*Cot[e + f*x])/(3*f) + (a^2*c*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*c*Csc
[(e + f*x)/2]^4)/(64*f) - (a^2*c*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (a^2*
c*Log[Cos[(e + f*x)/2]])/(8*f) - (a^2*c*Log[Sin[(e + f*x)/2]])/(8*f) - (a^2
*c*Sec[(e + f*x)/2]^2)/(32*f) + (a^2*c*Sec[(e + f*x)/2]^4)/(64*f)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

method	result
parallelrisch	$-\frac{a^2 c\left(-3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-8\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+24\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+24\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-24\left(\tan^3\left(\right.\right.\right.}{192 f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}$
derivativedivides	$\frac{a^2c\cot(fx+e)-a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+a^2c\left(\left(\left(-\right.\right.\right.}{f}$
default	$\frac{a^2c\cot(fx+e)-a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+a^2c\left(\left(\left(-\right.\right.\right.}{f}$
risch	$-\frac{a^2c\left(3\,e^{7i(fx+e)}+21\,e^{5i(fx+e)}-24i e^{6i(fx+e)}+21\,e^{3i(fx+e)}+24i e^{4i(fx+e)}+3\,e^{i(fx+e)}-8i e^{2i(fx+e)}+8i\right)}{12f\left(e^{2i(fx+e)}-1\right)^4}+\frac{a^2c\ln\left(e^{i(}\right.}{8}$
norman	$\frac{-\frac{a^2c}{64f}-\frac{a^2c\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{3a^2c\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{5a^2c\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{24f}-\frac{3a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64f}+\frac{a^2c\left(\tan^5\right.}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(1+\tan^2\left(\frac{fx}{2}+\right.\right.}$

```
[In] int(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/192*a^2*c*(-3*tan(1/2*f*x+1/2*e)^8-8*tan(1/2*f*x+1/2*e)^7+24*ln(tan(1/2*
f*x+1/2*e))*tan(1/2*f*x+1/2*e)^4+24*tan(1/2*f*x+1/2*e)^5-24*tan(1/2*f*x+1/2
*e)^3+8*tan(1/2*f*x+1/2*e)+3)/f/tan(1/2*f*x+1/2*e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.93

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{16\,a^2c\cos\left(fx+e\right)^3\sin\left(fx+e\right)+6\,a^2c\cos\left(fx+e\right)^3+6\,a^2c\cos\left(fx+e\right)-3\left(a^2c\cos\left(fx+e\right)^4-2\,a^2\right.}{48\left(f\cos\left(f\right.\right.}$$

```
[In] integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(16*a^2*c*cos(f*x + e)^3*sin(f*x + e) + 6*a^2*c*cos(f*x + e)^3 + 6*a^
2*c*cos(f*x + e) - 3*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c
)*log(1/2*cos(f*x + e) + 1/2) + 3*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x +
 e)^2 + a^2*c)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^4 - 2*f*cos(f*
x + e)^2 + f)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(78) = 156$.

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{3\,a^2c\left(\frac{2\left(3\,\cos(fx+e)^3-5\,\cos(fx+e)\right)}{\cos(fx+e)^4-2\,\cos(fx+e)^2+1} - 3\,\log\left(\cos\left(fx+e\right)+1\right) + 3\,\log\left(\cos\left(fx+e\right)-1\right)\right) - 12\,a^2c\left(\frac{2\,\cos(fx+e)}{\cos(fx+e)^2-1}\right.}{48\,f}$$

```
[In] integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
[Out] 1/48*(3*a^2*c*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*co
s(f*x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) - 12
*a^2*c*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(c
os(f*x + e) - 1)) + 48*a^2*c/tan(f*x + e) - 16*(3*tan(f*x + e)^2 + 1)*a^2*c
/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.63

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 8\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 24\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right) - 24\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{192\,f}$$

```
[In] integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
[Out] 1/192*(3*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 8*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 24
*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 24*a^2*c*tan(1/2*f*x + 1/2*e) + (50
*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 24*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 8*a^2*c*t
an(1/2*f*x + 1/2*e) - 3*a^2*c)/tan(1/2*f*x + 1/2*e)^4)/f
```

Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \csc^5(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24\,f}-\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8\,f}$$
$$-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(-2\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\frac{2\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{3}+\frac{c\,a^2}{4}\right)}{16\,f}$$
$$+\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}-\frac{a^2\,c\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^5,x)
```

```
[Out] (a^2*c*tan(e/2 + (f*x)/2)^3)/(24*f) - (a^2*c*tan(e/2 + (f*x)/2))/(8*f) - (c
ot(e/2 + (f*x)/2)^4*((a^2*c)/4 + (2*a^2*c*tan(e/2 + (f*x)/2))/3 - 2*a^2*c*t
an(e/2 + (f*x)/2)^3))/(16*f) + (a^2*c*tan(e/2 + (f*x)/2)^4)/(64*f) - (a^2*c
*log(tan(e/2 + (f*x)/2)))/(8*f)
```

3.10 $\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 105

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{8f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot^5(e+fx)}{5f}+\frac{a^2c\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f}$$

```
[Out] 1/8*a^2*c*arctanh(cos(f*x+e))/f-1/3*a^2*c*cot(f*x+e)^3/f-1/5*a^2*c*cot(f*x+
e)^5/f+1/8*a^2*c*cot(f*x+e)*csc(f*x+e)/f-1/4*a^2*c*cot(f*x+e)*csc(f*x+e)^3/
f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3045, 3853, 3855, 3852}

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{8f}-\frac{a^2c\cot^5(e+fx)}{5f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f}+\frac{a^2c\cot(e+fx)\csc(e+fx)}{8f}$$

```
[In] Int[Csc[e + f*x]^6*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Cos[e + f*x]])/(8*f) - (a^2*c*Cot[e + f*x]^3)/(3*f) - (a^2*c
*Cot[e + f*x]^5)/(5*f) + (a^2*c*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (a^2*c*C
ot[e + f*x]*Csc[e + f*x]^3)/(4*f)
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(-a^2c\csc^3(e+fx) - a^2c\csc^4(e+fx) + a^2c\csc^5(e+fx) + a^2c\csc^6(e+fx)\right)\,dx$$

$$= -\left(\left(a^2c\right)\int \csc^3(e+fx)\,dx\right) - \left(a^2c\right)\int \csc^4(e+fx)\,dx + \left(a^2c\right)\int \csc^5(e+fx)\,dx + \left(a^2c\right)\int \csc^6(e+fx)\,dx$$

$$= \frac{a^2c\cot(e+fx)\csc(e+fx)}{2f} - \frac{a^2c\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{1}{2}\left(a^2c\right)\int \csc(e+fx)\,dx + \frac{1}{4}\left(3a^2c\right)\int \csc^3(e+fx)\,dx + \frac{\left(a^2c\right)\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(e+fx)\right)}{f} - \frac{\left(a^2c\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(e+fx)\right)}{f}$$

$$= \frac{a^2 c \operatorname{arctanh}(\cos(e+fx))}{2f} - \frac{a^2 c \cot^3(e+fx)}{3f} - \frac{a^2 c \cot^5(e+fx)}{5f} + \frac{a^2 c \cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2 c \cot(e+fx)\csc^3(e+fx)}{4f} + \frac{1}{8}(3a^2 c)\int \csc(e+fx)\,dx$$

$$= \frac{a^2 c \operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{a^2 c \cot^3(e+fx)}{3f} - \frac{a^2 c \cot^5(e+fx)}{5f} + \frac{a^2 c \cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2 c \cot(e+fx)\csc^3(e+fx)}{4f}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.94

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{2a^2 c \cot(e+fx)}{15f} + \frac{a^2 c \csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} - \frac{a^2 c \csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{a^2 c \cot(e+fx)\csc^2(e+fx)}{15f} - \frac{a^2 c \cot(e+fx)\csc^4(e+fx)}{5f} + \frac{a^2 c \log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{a^2 c \log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} - \frac{a^2 c \sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a^2 c \sec^4\left(\frac{1}{2}(e+fx)\right)}{64f}$$

```
[In] Integrate[Csc[e + f*x]^6*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2*a^2*c*Cot[e + f*x])/(15*f) + (a^2*c*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*c*
Csc[(e + f*x)/2]^4)/(64*f) + (a^2*c*Cot[e + f*x]*Csc[e + f*x]^2)/(15*f) - (
a^2*c*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (a^2*c*Log[Cos[(e + f*x)/2]])/(8
*f) - (a^2*c*Log[Sin[(e + f*x)/2]])/(8*f) - (a^2*c*Sec[(e + f*x)/2]^2)/(32*
f) + (a^2*c*Sec[(e + f*x)/2]^4)/(64*f)
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17

method	result
parallelrisch	$-\frac{a^2c\left(-6\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6\left(\cot^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-15\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15\left(\cot^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-10\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+10\left(\cot^3\right.\right.}{960f}$
risch	$-\frac{a^2c\left(15\,\mathrm{e}^{9i(fx+e)}+240i\mathrm{e}^{6i(fx+e)}+90\,\mathrm{e}^{7i(fx+e)}+80i\mathrm{e}^{4i(fx+e)}+80i\mathrm{e}^{2i(fx+e)}-90\,\mathrm{e}^{3i(fx+e)}-16i-15\,\mathrm{e}^{i(fx+e)}\right)}{60f\left(\mathrm{e}^{2i(fx+e)}-1\right)^5}-\frac{a^2c}{}$
derivativedivides	$\frac{-a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+a^2c\left(\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\right.\right.\right.}{f}$
default	$\frac{-a^2c\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)-a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)+a^2c\left(\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\right.\right.\right.}{f}$
norman	$\frac{-\frac{a^2c}{160f}-\frac{a^2c\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{3a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{5a^2c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}-\frac{7a^2c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{240f}-\frac{3a^2c\left(\mathrm{ta}\right.}{}}{}$

```
[In] int(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/960*a^2*c*(-6*tan(1/2*f*x+1/2*e)^5+6*cot(1/2*f*x+1/2*e)^5-15*tan(1/2*f*x
+1/2*e)^4+15*cot(1/2*f*x+1/2*e)^4-10*tan(1/2*f*x+1/2*e)^3+10*cot(1/2*f*x+1/
2*e)^3+120*ln(tan(1/2*f*x+1/2*e))+60*tan(1/2*f*x+1/2*e)-60*cot(1/2*f*x+1/2*
e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.91

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{32\,a^2c\cos\left(fx+e\right)^5-80\,a^2c\cos\left(fx+e\right)^3+15\,\left(a^2c\cos\left(fx+e\right)^4-2\,a^2c\cos\left(fx+e\right)^2+a^2c\right)\log\left(\frac{1}{2}\,\cos\left(\right.\right.}{}$$

```
[In] integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(32*a^2*c*cos(f*x + e)^5 - 80*a^2*c*cos(f*x + e)^3 + 15*(a^2*c*cos(f*
x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c)*log(1/2*cos(f*x + e) + 1/2)*sin(
f*x + e) - 15*(a^2*c*cos(f*x + e)^4 - 2*a^2*c*cos(f*x + e)^2 + a^2*c)*log(-
1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 30*(a^2*c*cos(f*x + e)^3 + a^2*c*cos
(f*x + e))*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f
*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.78

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{15\,a^2c\left(\frac{2\left(3\,\cos(fx+e)^3-5\,\cos(fx+e)\right)}{\cos(fx+e)^4-2\,\cos(fx+e)^2+1}-3\,\log\left(\cos\left(fx+e\right)+1\right)+3\,\log\left(\cos\left(fx+e\right)-1\right)\right)-60\,a^2c\left(\frac{2\,\cos(fx+e}{\cos(fx+e)^2-}\right.}{2}$$

```
[In] integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
[Out] 1/240*(15*a^2*c*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*
cos(f*x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) -
60*a^2*c*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log
(cos(f*x + e) - 1)) + 80*(3*tan(f*x + e)^2 + 1)*a^2*c/tan(f*x + e)^3 - 16*(
15*tan(f*x + e)^4 + 10*tan(f*x + e)^2 + 3)*a^2*c/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.66

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{6\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+15\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-120\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.\right.}{96}$$

```
[In] integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/960*(6*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 15*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 1
0*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 120*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) -
 60*a^2*c*tan(1/2*f*x + 1/2*e) + (274*a^2*c*tan(1/2*f*x + 1/2*e)^5 + 60*a^2
*c*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 15*a^2*c*tan(
1/2*f*x + 1/2*e) - 6*a^2*c)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 12.00 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

$$\int \csc^6(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$

$$-\frac{a^2\,c\,\left(6\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-6\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-15\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+15\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\,\sin\left(\frac{e}{2}+\right.\right.}{}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^6,x)
```

```
[Out] -(a^2*c*(6*cos(e/2 + (f*x)/2)^10 - 6*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (
f*x)/2)*sin(e/2 + (f*x)/2)^9 + 15*cos(e/2 + (f*x)/2)^9*sin(e/2 + (f*x)/2) -
 10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^8 + 60*cos(e/2 + (f*x)/2)^4*sin
(e/2 + (f*x)/2)^6 - 60*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 10*cos(e
/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^2 + 120*cos(e/2 + (f*x)/2)^5*log(sin(e/2
 + (f*x)/2)/cos(e/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^5))/(960*f*cos(e/2 + (f*
x)/2)^5*sin(e/2 + (f*x)/2)^5)
```

3.11 $\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 130

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \frac{a^2c\operatorname{arctanh}(\cos(e+fx))}{16f}-\frac{a^2c\cot^3(e+fx)}{3f}-\frac{a^2c\cot^5(e+fx)}{5f}+\frac{a^2c\cot(e+fx)\csc(e+fx)}{16f}+\frac{a^2c\cot(e+fx)\csc^3(e+fx)}{24f}-\frac{a^2c\cot(e+fx)\csc^5(e+fx)}{6f}$$

```
[Out] 1/16*a^2*c*arctanh(cos(f*x+e))/f-1/3*a^2*c*cot(f*x+e)^3/f-1/5*a^2*c*cot(f*x
+e)^5/f+1/16*a^2*c*cot(f*x+e)*csc(f*x+e)/f+1/24*a^2*c*cot(f*x+e)*csc(f*x+e)
^3/f-1/6*a^2*c*cot(f*x+e)*csc(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {3045, 3852, 3853, 3855}

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$= \frac{a^2c\text{arctanh}(\cos(e+fx))}{16f} - \frac{a^2c\cot^5(e+fx)}{5f} - \frac{a^2c\cot^3(e+fx)}{3f} - \frac{a^2c\cot(e+fx)\csc^5(e+fx)}{6f} + \frac{a^2c\cot(e+fx)\csc^3(e+fx)}{24f} + \frac{a^2c\cot(e+fx)\csc(e+fx)}{16f}$$

```
[In] Int[Csc[e + f*x]^7*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Cos[e + f*x]])/(16*f) - (a^2*c*Cot[e + f*x]^3)/(3*f) - (a^2*
c*Cot[e + f*x]^5)/(5*f) + (a^2*c*Cot[e + f*x]*Csc[e + f*x])/(16*f) + (a^2*c
*Cot[e + f*x]*Csc[e + f*x]^3)/(24*f) - (a^2*c*Cot[e + f*x]*Csc[e + f*x]^5)/
(6*f)
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-a^2 c \csc^4(e+fx) - a^2 c \csc^5(e+fx) + a^2 c \csc^6(e+fx) + a^2 c \csc^7(e+fx)\right)\, dx \\
&= -\left(\left(a^2 c\right) \int \csc^4(e+fx)\, dx\right) - \left(a^2 c\right) \int \csc^5(e+fx)\, dx \\
&\quad + \left(a^2 c\right) \int \csc^6(e+fx)\, dx + \left(a^2 c\right) \int \csc^7(e+fx)\, dx \\
&= \frac{a^2 c \cot(e+fx) \csc^3(e+fx)}{4f} - \frac{a^2 c \cot(e+fx) \csc^5(e+fx)}{6f} \\
&\quad - \frac{1}{4}\left(3a^2 c\right) \int \csc^3(e+fx)\, dx + \frac{1}{6}\left(5a^2 c\right) \int \csc^5(e+fx)\, dx \\
&\quad + \frac{\left(a^2 c\right) \operatorname{Subst}\left(\int \left(1+x^2\right)\, dx, x, \cot(e+fx)\right)}{f} \\
&\quad - \frac{\left(a^2 c\right) \operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\, dx, x, \cot(e+fx)\right)}{f} \\
&= -\frac{a^2 c \cot^3(e+fx)}{3f} - \frac{a^2 c \cot^5(e+fx)}{5f} + \frac{3a^2 c \cot(e+fx) \csc(e+fx)}{8f} \\
&\quad + \frac{a^2 c \cot(e+fx) \csc^3(e+fx)}{24f} - \frac{a^2 c \cot(e+fx) \csc^5(e+fx)}{6f} \\
&\quad - \frac{1}{8}\left(3a^2 c\right) \int \csc(e+fx)\, dx + \frac{1}{8}\left(5a^2 c\right) \int \csc^3(e+fx)\, dx \\
&= \frac{3a^2 c \operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{a^2 c \cot^3(e+fx)}{3f} - \frac{a^2 c \cot^5(e+fx)}{5f} \\
&\quad + \frac{a^2 c \cot(e+fx) \csc(e+fx)}{16f} + \frac{a^2 c \cot(e+fx) \csc^3(e+fx)}{24f} \\
&\quad - \frac{a^2 c \cot(e+fx) \csc^5(e+fx)}{6f} + \frac{1}{16}\left(5a^2 c\right) \int \csc(e+fx)\, dx \\
&= \frac{a^2 c \operatorname{arctanh}(\cos(e+fx))}{16f} - \frac{a^2 c \cot^3(e+fx)}{3f} \\
&\quad - \frac{a^2 c \cot^5(e+fx)}{5f} + \frac{a^2 c \cot(e+fx) \csc(e+fx)}{16f} \\
&\quad + \frac{a^2 c \cot(e+fx) \csc^3(e+fx)}{24f} - \frac{a^2 c \cot(e+fx) \csc^5(e+fx)}{6f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.57

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{2a^2c\cot(e+fx)}{15f}+\frac{a^2c\csc^2\left(\frac{1}{2}(e+fx)\right)}{64f}-\frac{a^2c\csc^6\left(\frac{1}{2}(e+fx)\right)}{384f}+\frac{a^2c\cot(e+fx)\csc^2(e+fx)}{15f}-\frac{a^2c\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{a^2c\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{16f}-\frac{a^2c\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{16f}-\frac{a^2c\sec^2\left(\frac{1}{2}(e+fx)\right)}{64f}+\frac{a^2c\sec^6\left(\frac{1}{2}(e+fx)\right)}{384f}$$

```
[In] Integrate[Csc[e + f*x]^7*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2*a^2*c*Cot[e + f*x])/(15*f) + (a^2*c*Csc[(e + f*x)/2]^2)/(64*f) - (a^2*c*
Csc[(e + f*x)/2]^6)/(384*f) + (a^2*c*Cot[e + f*x]*Csc[e + f*x]^2)/(15*f) -
(a^2*c*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (a^2*c*Log[Cos[(e + f*x)/2]])/(
16*f) - (a^2*c*Log[Sin[(e + f*x)/2]])/(16*f) - (a^2*c*Sec[(e + f*x)/2]^2)/(
64*f) + (a^2*c*Sec[(e + f*x)/2]^6)/(384*f)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{19c\left(\frac{512\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{19}+\left(\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{17\cos(3fx+3e)}{114}-\frac{\cos(5fx+5e)}{38}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{128\cos(fx+e)}{57}+\cdots\right.\right.}{8192f}$
risch	$-\frac{a^2c\left(15\,\mathrm{e}^{11i(fx+e)}-85\,\mathrm{e}^{9i(fx+e)}-570\,\mathrm{e}^{7i(fx+e)}+480i\mathrm{e}^{8i(fx+e)}-570\,\mathrm{e}^{5i(fx+e)}-320i\mathrm{e}^{6i(fx+e)}-85\,\mathrm{e}^{3i(fx+e)}+15\,\mathrm{e}^{i(fx\cdots}\right.}{120f\left(\mathrm{e}^{2i(fx+e)}-1\right)^6}$
derivativedivides	$\frac{-a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)-a^2c\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+\cdots}{\cdots}$
default	$\frac{-a^2c\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)-a^2c\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+\cdots}{\cdots}$

```
[In] int(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -19/8192*c*(512/19*ln(tan(1/2*f*x+1/2*e))+(sec(1/2*f*x+1/2*e)*(cos(f*x+e)+1
7/114*cos(3*f*x+3*e)-1/38*cos(5*f*x+5*e))*csc(1/2*f*x+1/2*e)+128/57*cos(f*x
```

```
+e)+32/57*cos(3*f*x+3*e)-32/285*cos(5*f*x+5*e))*sec(1/2*f*x+1/2*e)^5*csc(1/
2*f*x+1/2*e)^5)*a^2/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$
$$-\frac{30\,a^2c\cos(fx+e)^5-80\,a^2c\cos(fx+e)^3-30\,a^2c\cos(fx+e)-15\left(a^2c\cos(fx+e)^6-3\,a^2c\cos(fx+e)^6-3\,a^2c\cos(fx+\right.}{}$$

```
[In] integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -1/480*(30*a^2*c*cos(f*x + e)^5 - 80*a^2*c*cos(f*x + e)^3 - 30*a^2*c*cos(f*
x + e) - 15*(a^2*c*cos(f*x + e)^6 - 3*a^2*c*cos(f*x + e)^4 + 3*a^2*c*cos(f*
x + e)^2 - a^2*c)*log(1/2*cos(f*x + e) + 1/2) + 15*(a^2*c*cos(f*x + e)^6 -
3*a^2*c*cos(f*x + e)^4 + 3*a^2*c*cos(f*x + e)^2 - a^2*c)*log(-1/2*cos(f*x +
 e) + 1/2) + 32*(2*a^2*c*cos(f*x + e)^5 - 5*a^2*c*cos(f*x + e)^3)*sin(f*x +
 e))/(f*cos(f*x + e)^6 - 3*f*cos(f*x + e)^4 + 3*f*cos(f*x + e)^2 - f)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**7*(a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$
$$=\frac{5\,a^2c\left(\frac{2\left(15\cos(fx+e)^5-40\cos(fx+e)^3+33\cos(fx+e)\right)}{\cos(fx+e)^6-3\cos(fx+e)^4+3\cos(fx+e)^2-1}-15\log\left(\cos(fx+e)+1\right)+15\log\left(\cos(fx+e)-1\right)\right)-30}{}$$

```
[In] integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(5*a^2*c*(2*(15*cos(f*x + e)^5 - 40*cos(f*x + e)^3 + 33*cos(f*x + e))
/(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1) - 15*log(cos(f*
x + e) + 1) + 15*log(cos(f*x + e) - 1)) - 30*a^2*c*(2*(3*cos(f*x + e)^3 - 5
*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1) - 3*log(cos(f*x + e)
 + 1) + 3*log(cos(f*x + e) - 1)) + 160*(3*tan(f*x + e)^2 + 1)*a^2*c/tan(f*x
 + e)^3 - 32*(15*tan(f*x + e)^4 + 10*tan(f*x + e)^2 + 3)*a^2*c/tan(f*x + e)
^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(118) = 236$.

Time = 0.38 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.86

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$$

$$= \frac{5\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+12\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+15\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+20\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-}{}$$

```
[In] integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/1920*(5*a^2*c*tan(1/2*f*x + 1/2*e)^6 + 12*a^2*c*tan(1/2*f*x + 1/2*e)^5 +
15*a^2*c*tan(1/2*f*x + 1/2*e)^4 + 20*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*
c*tan(1/2*f*x + 1/2*e)^2 - 120*a^2*c*log(abs(tan(1/2*f*x + 1/2*e))) - 120*a
^2*c*tan(1/2*f*x + 1/2*e) + (294*a^2*c*tan(1/2*f*x + 1/2*e)^6 + 120*a^2*c*t
an(1/2*f*x + 1/2*e)^5 + 15*a^2*c*tan(1/2*f*x + 1/2*e)^4 - 20*a^2*c*tan(1/2*
f*x + 1/2*e)^3 - 15*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*c*tan(1/2*f*x + 1
/2*e) - 5*a^2*c)/tan(1/2*f*x + 1/2*e)^6)/f
```

Mupad [B] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.62

$$\int \csc^7(e+fx)(a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx =$$

$$-\frac{a^2\,c\left(5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-12\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+12\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}\,\sin\left(\frac{e}{2}\right.\right.}{}$$

```
[In] int(((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)))/sin(e + f*x)^7,x)
```

```
[Out] -(a^2*c*(5*cos(e/2 + (f*x)/2)^12 - 5*sin(e/2 + (f*x)/2)^12 - 12*cos(e/2 + (
f*x)/2)*sin(e/2 + (f*x)/2)^11 + 12*cos(e/2 + (f*x)/2)^11*sin(e/2 + (f*x)/2)
 - 15*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^10 - 20*cos(e/2 + (f*x)/2)^3*
sin(e/2 + (f*x)/2)^9 + 15*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^8 + 120*c
os(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7 - 120*cos(e/2 + (f*x)/2)^7*sin(e/2
 + (f*x)/2)^5 - 15*cos(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^4 + 20*cos(e/2 +
 (f*x)/2)^9*sin(e/2 + (f*x)/2)^3 + 15*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)
/2)^2 + 120*cos(e/2 + (f*x)/2)^6*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))
*sin(e/2 + (f*x)/2)^6))/(1920*f*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6)
```

3.12 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx$

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Rubi [A] (verified) 105
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Optimal result

Integrand size = 34, antiderivative size = 128

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx = -\frac{8a^3c\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a^2c\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}+\frac{4ac\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2c\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}$$

```
[Out] -8/63*a^3*c*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*c*cos(d*x+c)^3*(a+a*s
in(d*x+c))^(3/2)/d-2/21*a^2*c*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+4/21*a*
c*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3055, 3060, 2838, 2830, 2725}

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx = \frac{2a^2c\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2c\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2ac\sin^3(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{9d}-\frac{2c\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{21d}+\frac{4ac\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{63d}$$

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2)*(c - c*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*c*Cos[c + d*x])/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a^2*c*Cos[c + d
*x]*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) + (4*a*c*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(63*d) + (2*a*c*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a
 + a*Sin[c + d*x]])/(9*d) - (2*c*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(
21*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ac\cos(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}}{9d} \\
&\quad+\frac{2}{9}\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\left(\frac{3ac}{2}-\frac{1}{2}ac\sin(c+dx)\right)\,dx \\
&= \frac{2a^2c\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}+\frac{2ac\cos(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}}{9d} \\
&\quad+\frac{1}{21}(5ac)\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx \\
&= \frac{2a^2c\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}+\frac{2ac\cos(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}}{9d} \\
&\quad-\frac{2c\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{21d} \\
&\quad+\frac{1}{21}(2c)\int \left(\frac{3a}{2}-a\sin(c+dx)\right)\sqrt{a+a\sin(c+dx)}\,dx \\
&= \frac{2a^2c\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}+\frac{4ac\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{63d} \\
&\quad+\frac{2ac\cos(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}}{9d} \\
&\quad-\frac{2c\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{21d}+\frac{1}{9}(ac)\int \sqrt{a+a\sin(c+dx)}\,dx \\
&= -\frac{2a^2c\cos(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{2a^2c\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} \\
&\quad+\frac{4ac\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{63d} \\
&\quad+\frac{2ac\cos(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}}{9d} \\
&\quad-\frac{2c\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{21d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx =$$
$$-\frac{2ac\sec(c+dx)(-1+\sin(c+dx))^2\sqrt{a(1+\sin(c+dx))}(8+12\sin(c+dx)+15\sin^2(c+dx)+7\sin^3(c+d}{63d}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2)*(c - c*Sin[c + d*x]),x]
[Out] (-2*a*c*Sec[c + d*x]*(-1 + Sin[c + d*x])^2*Sqrt[a*(1 + Sin[c + d*x])]*(8 +
12*Sin[c + d*x] + 15*Sin[c + d*x]^2 + 7*Sin[c + d*x]^3))/(63*d)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

method	result
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2c(7(\sin^3(dx+c))+15(\sin^2(dx+c))+12\sin(dx+c)+8)}{63\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$
parts	$\frac{2c(1+\sin(dx+c))a^2(\sin(dx+c)-1)(15(\sin^3(dx+c))+39(\sin^2(dx+c))+52\sin(dx+c)+104)}{105\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}-\frac{2c(1+\sin(dx+c))a^2(\sin(dx+c)-1)(35(}{3}$

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x,method=_RETURNVE
RBOSE)
[Out] -2/63*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*c*(7*sin(d*x+c)^3+15*sin(d*x+c)^2
+12*sin(d*x+c)+8)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c$$
$$-c\sin(c+dx))\,dx = \frac{2\left(7\,ac\cos\left(dx+c\right)^5-ac\cos\left(dx+c\right)^4-11\,ac\cos\left(dx+c\right)^3+ac\cos\left(dx+c\right)^2-4\,ac\cos}{}$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm
="fricas")
[Out] 2/63*(7*a*c*cos(d*x + c)^5 - a*c*cos(d*x + c)^4 - 11*a*c*cos(d*x + c)^3 + a
*c*cos(d*x + c)^2 - 4*a*c*cos(d*x + c) - 8*a*c - (7*a*c*cos(d*x + c)^4 + 8*
a*c*cos(d*x + c)^3 - 3*a*c*cos(d*x + c)^2 - 4*a*c*cos(d*x + c) - 8*a*c)*sin
(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx = -c\left(\int \left(-a\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)\right)\,dx + \int a\sqrt{a\sin(c+dx)+a}\sin^4(c+dx)\,dx\right)$$

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2)*(c-c*sin(d*x+c)),x)
[Out] -c*(Integral(-a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**2, x) + Integral(a*s
qrt(a*sin(c + d*x) + a)*sin(c + d*x)**4, x))
```

Maxima [F]

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx = \int -(a\sin(dx+c)+a)^{\frac{3}{2}}(c\sin(dx+c)-c)\sin(dx+c)^2\,dx$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm
="maxima")
[Out] -integrate((a*sin(d*x + c) + a)^(3/2)*(c*sin(d*x + c) - c)*sin(d*x + c)^2,
x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx = \frac{\sqrt{2}\left(126\,ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-9\,ac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.\right.}{}$$

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)*(c-c*sin(d*x+c)),x, algorithm
="giac")
[Out] 1/504*sqrt(2)*(126*a*c*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) - 9*a*c*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/
2*d*x + 7/2*c) - 7*a*c*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9/4*pi + 9/
2*d*x + 9/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}(c-c\sin(c+dx))\,dx=\int \sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(c-c\,\sin\left(c+d\,x\right)\right)\,dx$$

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)*(c - c*sin(c + d*x)),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)*(c - c*sin(c + d*x)), x)
```

3.13 $\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx$

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Fricas [B] (verification not implemented) 113
Sympy [F] 114
Maxima [F] 114
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Mupad [F(-1)] 115

Optimal result

Integrand size = 34, antiderivative size = 69

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{cf}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/c/f+2*sec(f*x
+e)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3013, 2852, 212, 2815, 2752}

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{cf} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{cf}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(c*f)
 + (2*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3013

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*((c
_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b
*Sin[e + f*x]]/Sin[e + f*x], x], x] - Dist[d/c, Int[Sqrt[a + b*Sin[e + f*x]
]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{c} + \int \frac{\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx$$

$$= \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx}{ac} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf}$$

$$= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{cf}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sin(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sin(e+fx))}}{cf}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sin[e + f*x]]*Sec[e + f*x]*Sqrt[a*(1
 + Sin[e + f*x])])/(c*f)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{2(1+\sin(fx+e))\left(a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right)a\sqrt{a-a\sin(fx+e)}\right)}{\sqrt{a}\,c\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	78

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*(1+sin(f*x+e))*(a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a*(a-a*si
n(f*x+e))^(1/2))/a^(1/2)/c/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(61) = 122.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.93

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{\sqrt{a}\cos(fx+e)\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos}\right.}{2\,cf\cos(fx+e)}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] 1/2*(sqrt(a)*cos(f*x + e)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(c
os(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(
a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*co
s(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f
*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) +
a))/(c*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = -\frac{\int \frac{\sqrt{a\sin(e+fx)+a}}{\sin^2(e+fx)-\sin(e+fx)}\,dx}{c}$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x)
[Out] -Integral(sqrt(a*sin(e + f*x) + a)/(sin(e + f*x)**2 - sin(e + f*x)), x)/c
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \int -\frac{\sqrt{a\sin(fx+e)+a}}{(c\sin(fx+e)-c)\sin(fx+e)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="
maxima")
[Out] -integrate(sqrt(a*sin(f*x + e) + a)/((c*sin(f*x + e) - c)*sin(f*x + e)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c}+\frac{2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}\right)\sqrt{a}}{2f}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))/c + 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(c*sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx=\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c - c*sin(e + f*x))),x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c - c*sin(e + f*x))), x)
```

3.14 $\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 34, antiderivative size = 120

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{acf}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*arcta
nh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/
2)+sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3019, 2815, 2752, 3064, 2728, 212, 2852}

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}cf} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{acf}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(Sqrt[a]*c*f)
 + ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sqrt
[2]*Sqrt[a]*c*f) + (Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(a*c*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2815

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3019

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[d^2/(c*(b*c - a*d
)), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] + Dist[1/(c*(
b*c - a*d)), Int[(b*c - a*d - b*d*Sin[e + f*x])/(Sin[e + f*x]*Sqrt[a + b*Si
n[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx}{2a} + \frac{\int \frac{\csc(e+fx)(2ac+ac\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx}{2ac^2} \\
&= -\frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{2c} + \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx}{2a^2c} \\
&\quad + \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{ac} \\
&= \frac{\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{acf} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} \\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{acf}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sin(e+fx)\right)\right.}{acf}$$

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])),x]
```

```
[Out] -(((Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sin[e + f*x])/2] - 2*Hypergeometri
c2F1[-1/2, 1, 1/2, 1 - Sin[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x]
)])/(a*c*f))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

method	result
default	$\frac{(1+\sin(fx+e))\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\sqrt{a-a\sin(fx+e)}+2a^{\frac{5}{2}}-4\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right)a^{2}\sqrt{a-a\sin(fx+e)}\right)}{2c\,a^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

```
[In] int(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*(1+sin(f*x+e))*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*(a-a*sin(f*x+e))^(1/2)+2*a^(5/2)-4*arctanh((a-a*sin(f*x+e))^(1/2)/
a^(1/2))*a^2*(a-a*sin(f*x+e))^(1/2))/c/a^(5/2)/cos(f*x+e)/(a+a*sin(f*x+e))^
(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(104) = 208.

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.73

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\sqrt{2}\sqrt{a}\cos\left(fx+e\right)\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+}{}$$

```
[In] integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)
*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x
+ e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*sqrt(a)*cos(f*x + e)*log((a*cos(f*x
 + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f
*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*
x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f
*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x +
e) - 1)) + 4*sqrt(a*sin(f*x + e) + a))/(a*c*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\int \frac{1}{\sqrt{a\sin(e+fx)+a}\sin^2(e+fx)-\sqrt{a\sin(e+fx)+a}\sin(e+fx)}\,dx}{c}$$

```
[In] integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
[Out] -Integral(1/(sqrt(a*sin(e + f*x) + a)*sin(e + f*x)**2 - sqrt(a*sin(e + f*x)
 + a)*sin(e + f*x)), x)/c
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \int -\frac{1}{\sqrt{a\sin(fx+e)+a}(c\sin(fx+e)-c)\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="maxima")
[Out] -integrate(1/(sqrt(a*sin(f*x + e) + a)*(c*sin(f*x + e) - c)*sin(f*x + e)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{c}+\frac{\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{c}-\frac{\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{c}+\frac{2}{c\sin(-\frac{1}{4}\pi+\frac{1}{2}f}\right.}{4\sqrt{a}f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

```
[In] integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/4*sqrt(2)*(2*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*
e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c + log(sin(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1)/c - log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/c + 2/
(c*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sqrt(a)*f*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$$
$$=\int \frac{1}{\sin(e+f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}\,(c-c\,\sin(e+f\,x))}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))),x)
[Out] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))), x)
```

3.15 $\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx$

Optimal result 122
Rubi [A] (verified) 122
Mathematica [A] (verified) 124
Maple [B] (warning: unable to verify) 124
Fricas [A] (verification not implemented) 125
Sympy [F] 126
Maxima [F] 126
Giac [F(-1)] 127
Mupad [F(-1)] 127

Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{cf}$$

```
[Out] 2*arctan(cos(f*x+e)*a^(1/2)*g^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(
1/2))*a^(1/2)*g^(1/2)/c/f+2*sec(f*x+e)*(g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e)
)^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3007, 2854, 211, 3009, 12, 30}

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{cf} + \frac{2\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}{cf}$$

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[g]*ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[g*Sin[e + f*
x]]*Sqrt[a + a*Sin[e + f*x]])])/(c*f) + (2*Sec[e + f*x]*Sqrt[g*Sin[e + f*x]
]*Sqrt[a + a*Sin[e + f*x]])/(c*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3007

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]])/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g/d, In
t[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Sin[e + f*x]], x], x] - Dist[c*(g/d), Int
[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^
2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = g\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx - \frac{g\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{(2ag)\text{Subst}\left(\int \frac{1}{cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{(2ag)\text{Subst}\left(\int \frac{1}{a+gx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf}$$

$$= \frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf} - \frac{(2a)\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf}$$

$$= \frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{cf}$$

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \frac{2\sec(e+fx)\left(\arcsin\left(\sqrt{1-\sin(e+fx)}\right)\sqrt{1-\sin(e+fx)}+\sqrt{\sin(e+fx)}\right)\sqrt{g\sin(e+fx)}\sqrt{a(1+\sin}}{cf\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*
x]),x]
```

```
[Out] (2*Sec[e + f*x]*(ArcSin[Sqrt[1 - Sin[e + f*x]]]*Sqrt[1 - Sin[e + f*x]] + Sq
rt[Sin[e + f*x]])*Sqrt[g*Sin[e + f*x]]*Sqrt[a*(1 + Sin[e + f*x])])/(c*f*Sqr
t[Sin[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(87) = 174.

Time = 3.21 (sec) , antiderivative size = 754, normalized size of antiderivative = 7.32

method	result
default	$-\frac{\sqrt{\frac{g(\csc(fx+e)-\cot(fx+e))}{(1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1}}\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1\right)\sqrt{\frac{a\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+2\csc(fx+e)-2\cot(fx+e)+1\right)}{(1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1}}}{}$

```
[In] int((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/4/c/f*(g/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(csc(f*x+e)-cot(f*x+e)))^(1/2
)*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(a*((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc
(f*x+e)-2*cot(f*x+e)+1)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))^(1/2)/(-cot(f*x+
e)+csc(f*x+e)+1)*(2^(1/2)*ln(-(csc(f*x+e)-cot(f*x+e)+(csc(f*x+e)-cot(f*x+e)
)^(1/2)*2^(1/2)+1)/((csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)-csc(f*x+e)+cot(f*
x+e)-1))*(csc(f*x+e)-cot(f*x+e))+4*2^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(
1/2)*2^(1/2)+1)*(csc(f*x+e)-cot(f*x+e))+4*2^(1/2)*arctan((csc(f*x+e)-cot(f*
x+e))^(1/2)*2^(1/2)-1)*(csc(f*x+e)-cot(f*x+e))+2^(1/2)*ln(-((csc(f*x+e)-cot
(f*x+e))^(1/2)*2^(1/2)-csc(f*x+e)+cot(f*x+e)-1)/(csc(f*x+e)-cot(f*x+e)+(csc
(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1))*(csc(f*x+e)-cot(f*x+e))-2^(1/2)*ln(-(
csc(f*x+e)-cot(f*x+e)+(csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1)/((csc(f*x+e)
-cot(f*x+e))^(1/2)*2^(1/2)-csc(f*x+e)+cot(f*x+e)-1))-4*2^(1/2)*arctan((csc(
f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1)-4*2^(1/2)*arctan((csc(f*x+e)-cot(f*x+e)
)^(1/2)*2^(1/2)-1)-2^(1/2)*ln(-((csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)-csc(f
*x+e)+cot(f*x+e)-1)/(csc(f*x+e)-cot(f*x+e)+(csc(f*x+e)-cot(f*x+e))^(1/2)*2^
(1/2)+1))+8*(csc(f*x+e)-cot(f*x+e))^(1/2))/(csc(f*x+e)-cot(f*x+e))^(1/2)/((
csc(f*x+e)-cot(f*x+e))^(1/2)+1)/((csc(f*x+e)-cot(f*x+e))^(1/2)-1)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.29

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-ag}\cos(fx+e)\log\left(\frac{128\,ag\cos(fx+e)^5-128\,ag\cos(fx+e)^4-416\,ag\cos(fx+e)^3+128\,ag\cos(fx+e)^2+289\,ag\cos(fx+e)+8\left(16\,\mathrm{co}\right.}{}\right.}{}\right.$$

$$-\frac{\sqrt{ag}\arctan\left(\frac{\sqrt{ag}\left(8\cos(fx+e)^2+8\sin(fx+e)-9\right)\sqrt{a\sin(fx+e)+a}\sqrt{g\sin(fx+e)}}{4\left(2\,ag\cos(fx+e)^3+ag\cos(fx+e)\sin(fx+e)-2\,ag\cos(fx+e)\right)}\right)\cos(fx+e)-4\sqrt{a\sin(fx+e)+}}{2\,cf\cos(fx+e)}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a*g)*cos(f*x + e)*log((128*a*g*cos(f*x + e)^5 - 128*a*g*cos(f*x
 + e)^4 - 416*a*g*cos(f*x + e)^3 + 128*a*g*cos(f*x + e)^2 + 289*a*g*cos(f*x
 + e) + 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^3 - 66*cos(f*x + e)^2 + (16*
cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x + e) - 51)*sin(f*x + e) + 2
5*cos(f*x + e) + 51)*sqrt(-a*g)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e
)) + a*g + (128*a*g*cos(f*x + e)^4 + 256*a*g*cos(f*x + e)^3 - 160*a*g*cos(f
*x + e)^2 - 288*a*g*cos(f*x + e) + a*g)*sin(f*x + e))/(cos(f*x + e) + sin(f
*x + e) + 1)) + 8*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(c*f*cos(f
*x + e)), -1/2*(sqrt(a*g)*arctan(1/4*sqrt(a*g)*(8*cos(f*x + e)^2 + 8*sin(f*
x + e) - 9)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(2*a*g*cos(f*x +
e)^3 + a*g*cos(f*x + e)*sin(f*x + e) - 2*a*g*cos(f*x + e)))*cos(f*x + e) -
4*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = -\frac{\int \frac{\sqrt{g\sin{(e+fx)}}\sqrt{a\sin{(e+fx)}+a}}{\sin{(e+fx)}-1}\,dx}{c}$$

```
[In] integrate((g*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e)),x)
```

```
[Out] -Integral(sqrt(g*sin(e + f*x))*sqrt(a*sin(e + f*x) + a)/(sin(e + f*x) - 1),
 x)/c
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \int -\frac{\sqrt{a\sin{(fx+e)}+a}\sqrt{g\sin{(fx+e)}}}{c\sin{(fx+e)}-c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] -integrate(sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) -
c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c-c\sin(e+fx)}\,dx = \int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c-c\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
), x)
```

3.16 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx$

Optimal result 128
Rubi [A] (verified) 128
Mathematica [A] (verified) 129
Maple [A] (verified) 129
Fricas [A] (verification not implemented) 130
Sympy [F] 130
Maxima [B] (verification not implemented) 130
Giac [F] 131
Mupad [B] (verification not implemented) 131

Optimal result

Integrand size = 40, antiderivative size = 43

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{2\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{cfg}$$

```
[Out] 2*sec(f*x+e)*(g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {3009, 12, 30}

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{2\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}{cfg}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c - c*Sin[e + f*x])),x]
[Out] (2*Sec[e + f*x]*Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c*f*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{f}\\ &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cfg}\\ &= \frac{2\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{cfg}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{2\sqrt{a(1+\sin(e+fx))}\tan(e+fx)}{cf\sqrt{g\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c - c*Sin[e + f*x
])),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[e + f*x])]*Tan[e + f*x])/(c*f*Sqrt[g*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\tan(fx+e)\sqrt{a(1+\sin(fx+e))}}{cf\sqrt{g\sin(fx+e)}}$	37

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 2/c/f*tan(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(g*sin(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{2\sqrt{a\sin(fx+e)+a}\sqrt{g\sin(fx+e)}}{cfg\cos(fx+e)}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*f*g*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\int \frac{\sqrt{a\sin(e+fx)+a}}{\sqrt{g\sin(e+fx)}\sin(e+fx)-\sqrt{g\sin(e+fx)}}\,dx}{c}$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))/(g*sin(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sqrt(a*sin(e + f*x) + a)/(sqrt(g*sin(e + f*x))*sin(e + f*x) - sqr
t(g*sin(e + f*x))), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(39) = 78.

Time = 0.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 7.19

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\frac{4\left(\left(\frac{3\sqrt{2}\sqrt{a}\sqrt{g}\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sqrt{2}\sqrt{a}\sqrt{g}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}-\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sqrt{g}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{2}\sqrt{a}\sqrt{g}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}}\right)}{cg-\frac{cg\sin(fx+e)}{\cos(fx+e)+1}}-\frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+}{c\sqrt{g}}}{12f}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] -1/12*(4*((3*sqrt(2)*sqrt(a)*sqrt(g)*sin(f*x + e)/(cos(f*x + e) + 1) - sqrt
(2)*sqrt(a)*sqrt(g)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*sqrt(sin(f*x + e)/
(cos(f*x + e) + 1)) - 2*(3*sqrt(2)*sqrt(a)*sqrt(g)*sin(f*x + e)/(cos(f*x +
```

```
e) + 1) + sqrt(2)*sqrt(a)*sqrt(g)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/sqrt
(sin(f*x + e)/(cos(f*x + e) + 1)))/(c*g - c*g*sin(f*x + e)/(cos(f*x + e) +
1)) - (2*sqrt(2)*sqrt(a)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2) + 3*sqrt(2
)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1))/(c*sqrt(g)) - (2*sqrt(2)*sqrt(a)
*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2) - 3*sqrt(2)*sqrt(a)*sin(f*x + e)/(
cos(f*x + e) + 1))/(c*sqrt(g)))/f
```

Giac [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx=\int -\frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(c\sin\left(fx+e\right)-c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-sqrt(a*sin(f*x + e) + a)/((c*sin(f*x + e) - c)*sqrt(g*sin(f*x +
e))), x)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx=\frac{2\,\sin\left(2\,e+2\,f\,x\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}}{c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{g\,\sin\left(e+f\,x\right)}}$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
),x)
```

```
[Out] (2*sin(2*e + 2*f*x)*(a*(sin(e + f*x) + 1))^(1/2))/(c*f*(cos(2*e + 2*f*x) +
1)*(g*sin(e + f*x))^(1/2))
```

3.17 $\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$

Optimal result 132
Rubi [A] (verified) 132
Mathematica [A] (verified) 134
Maple [A] (verified) 134
Fricas [A] (verification not implemented) 135
Sympy [F] 136
Maxima [B] (verification not implemented) 136
Giac [F(-1)] 136
Mupad [F(-1)] 137

Optimal result

Integrand size = 40, antiderivative size = 114

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{acf}$$

```
[Out] 1/2*arctan(1/2*cos(f*x+e)*a^(1/2)*g^(1/2)*2^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a
*sin(f*x+e))^(1/2))*g^(1/2)/c/f*2^(1/2)/a^(1/2)+sec(f*x+e)*(g*sin(f*x+e))^(
1/2)*(a+a*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3015, 2861, 211, 3009, 12, 30}

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}{acf}$$

```
[In] Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])),x]
```

```
[Out] (Sqrt[g]*ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[2]*Sqrt[g*Sin[e + f*x]
]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*Sqrt[a]*c*f) + (Sec[e + f*x]*Sqrt[g*
Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(a*c*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3015

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-a)*(g
/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x
] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*
x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx}{2a} - \frac{g\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\,dx}{2c} \\
&= -\frac{g\text{Subst}\left(\int \frac{1}{cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{f} \\
&\quad + \frac{(ag)\text{Subst}\left(\int \frac{1}{2a^2+agx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf} \\
&= \frac{\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} - \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf} \\
&= \frac{\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{acf}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$
\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\csc(2(e+fx))\sqrt{\sin(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{a(1+\sin(e+fx))}\left(2\sqrt{c}\sqrt{\sin(e+fx)} - \sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{s}}{\sqrt{c-c\sin}}\right.\right.}{ac^{3/2}f}
$$

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
]])),x]
```

```
[Out] (Csc[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*Sqrt[g*Sin[e + f*x]]*Sqrt[a*(1 + Sin[e
 + f*x])]*(2*Sqrt[c]*Sqrt[Sin[e + f*x]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*S
qrt[Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]]]*Sqrt[c - c*Sin[e + f*x]]))/(a*
c^(3/2)*f)
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\left(\sqrt{\csc(fx+e)-\cot(fx+e)}\,\sin(fx+e)-2\arctan\left(\sqrt{\csc(fx+e)-\cot(fx+e)}\right)\cos(fx+e)+\sqrt{\csc(fx+e)-\cot(fx+e)}\,\cos(fx+e)+\sqrt{cs}\right.}{cf(-\cos(fx+e)+\sin(fx+e)-1)\sqrt{a(1+\sin(fx+e))}\,\sqrt{\csc(fx+e)-\cot(fx+e)}}$

```
[In] int((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/c/f*((csc(f*x+e)-cot(f*x+e))^(1/2)*sin(f*x+e)-2*arctan((csc(f*x+e)-cot(f
*x+e))^(1/2))*cos(f*x+e)+(csc(f*x+e)-cot(f*x+e))^(1/2)*cos(f*x+e)+(csc(f*x+
e)-cot(f*x+e))^(1/2))*(g*sin(f*x+e))^(1/2)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1
+sin(f*x+e)))^(1/2)/(csc(f*x+e)-cot(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{g}{a}}\cos(fx+e)\log\left(\frac{17\,g\cos(fx+e)^3+4\sqrt{2}\left(3\cos(fx+e)^2+(3\cos(fx+e)+4)\sin(fx+e)-\cos(fx+e)-4\right)\sqrt{a\sin(fx+e)+a}\sqrt{\cdots}}{\cos(fx+e)^3+3\cos(fx+e)^2+\left(\cos(fx+e)\cdots\right.}\right.}{\cdots}\right. - \frac{\sqrt{2}a\sqrt{\frac{g}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{g\sin(fx+e)}\sqrt{\frac{g}{a}}(3\sin(fx+e)-1)}{4\,g\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)-4\sqrt{a\sin(fx+e)+a}\sqrt{g\,s\cdots}}{4\,acf\cos(fx+e)}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*a*sqrt(-g/a)*cos(f*x + e)*log((17*g*cos(f*x + e)^3 + 4*sqrt(2
)*(3*cos(f*x + e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)
*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e
)^2 - 18*g*cos(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*s
in(f*x + e) - 4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2
*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + 8*sqrt(a*sin(f*x +
 e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*cos(f*x + e)), -1/4*(sqrt(2)*a*sqrt(g
/a)*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(g
/a)*(3*sin(f*x + e) - 1)/(g*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) - 4*sq
rt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a\sin(e+fx)+a}\sin(e+fx)-\sqrt{a\sin(e+fx)+a}}\,dx}{c}$$

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sqrt(g*sin(e + f*x))/(sqrt(a*sin(e + f*x) + a)*sin(e + f*x) - sqr
t(a*sin(e + f*x) + a)), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(96) = 192$.

Time = 0.32 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$$
$$= \frac{\frac{4\sqrt{2}\sqrt{g}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}-\frac{\left(\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}\right)\sin(fx+e)}{\cos(fx+e)+1}} - \frac{2\sqrt{2}\sqrt{g}\arctan\left(\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)}{\sqrt{a}c} + \frac{\sqrt{2}\sqrt{g}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}+\sqrt{2}\sqrt{g}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}} + \frac{\sqrt{2}\sqrt{g}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}-\sqrt{2}\sqrt{g}}{\sqrt{a}c+\frac{\sqrt{a}c\sin(fx+e)}{\cos(fx+e)+1}}}{2f}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*sqrt(g)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2)/(sqrt(a)*c +
 sqrt(a)*c*sin(f*x + e)/(cos(f*x + e) + 1) - (sqrt(a)*c + sqrt(a)*c*sin(f*x
 + e)/(cos(f*x + e) + 1))*sin(f*x + e)/(cos(f*x + e) + 1)) - 2*sqrt(2)*sqrt
(g)*arctan(sqrt(sin(f*x + e)/(cos(f*x + e) + 1)))/(sqrt(a)*c) + (sqrt(2)*sq
rt(g)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1)) + sqrt(2)*sqrt(g))/(sqrt(a)*c +
 sqrt(a)*c*sin(f*x + e)/(cos(f*x + e) + 1)) + (sqrt(2)*sqrt(g)*sqrt(sin(f*x
 + e)/(cos(f*x + e) + 1)) - sqrt(2)*sqrt(g))/(sqrt(a)*c + sqrt(a)*c*sin(f*x
 + e)/(cos(f*x + e) + 1)))/f
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$$
$$=\int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
),x)
[Out] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
), x)
```

3.18 $\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$

Optimal result . . . 138
Rubi [A] (verified) . . . 138
Mathematica [A] (verified) . . . 140
Maple [A] (verified) . . . 140
Fricas [A] (verification not implemented) . . . 141
Sympy [F] . . . 141
Maxima [F] . . . 142
Giac [F(-1)] . . . 142
Mupad [F(-1)] . . . 142

Optimal result

Integrand size = 40, antiderivative size = 118

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf\sqrt{g}}+\frac{\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{acfg}$$

```
[Out] -1/2*arctan(1/2*cos(f*x+e)*a^(1/2)*g^(1/2)*2^(1/2)/(g*sin(f*x+e))^(1/2)/(a+
a*sin(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)/g^(1/2)+sec(f*x+e)*(g*sin(f*x+e))^
(1/2)*(a+a*sin(f*x+e))^(1/2)/a/c/f/g
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3017, 2861, 211, 3009, 12, 30}

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}{acfg}-\frac{\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf\sqrt{g}}$$

```
[In] Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])),
x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[2]*Sqrt[g*Sin[e + f*x]]*Sqrt[
a + a*Sin[e + f*x]])]/(Sqrt[2]*Sqrt[a]*c*f*Sqrt[g])) + (Sec[e + f*x]*Sqrt[g
*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(a*c*f*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3017

```
Int[1/(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*
c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - D
ist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c +
d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*
d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c-c\sin(e+fx))}\,dx}{2a} + \frac{\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\,dx}{2c}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{f} - \frac{a\text{Subst}\left(\int \frac{1}{2a^2+agx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cf}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf\sqrt{g}} - \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{cfg}$$

$$= -\frac{\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf\sqrt{g}} + \frac{\sec(e+fx)\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{acfg}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \frac{\csc(2(e+fx))\sin^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sin(e+fx))}\left(2\sqrt{c}\sqrt{\sin(e+fx)}+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\right)\sqrt{c-c}\right.}{ac^{3/2}f\sqrt{g\sin(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])),x]
```

```
[Out] (Csc[2*(e + f*x)]*Sin[e + f*x]^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*(2*Sqrt[c]*
Sqrt[Sin[e + f*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sin[e + f*x]])/Sq
rt[c - c*Sin[e + f*x]]]*Sqrt[c - c*Sin[e + f*x]]))/(a*c^(3/2)*f*Sqrt[g*Sin[
e + f*x]])
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

method	result
default	$-\frac{\left(2\cos(fx+e)\sqrt{\csc(fx+e)-\cot(fx+e)}\,\arctan\left(\sqrt{\csc(fx+e)-\cot(fx+e)}\right)+1-\cos(fx+e)-\cos(fx+e)\cot(fx+e)+\csc(fx+e)\right)(1+c}{cf(-\cos(fx+e)+\sin(fx+e)-1)\sqrt{a(1+\sin(fx+e))}\,\sqrt{g\sin(fx+e)}}$

```
[In] int(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/c/f*(2*cos(f*x+e)*(csc(f*x+e)-cot(f*x+e))^(1/2)*arctan((csc(f*x+e)-cot(f
*x+e))^(1/2))+1-cos(f*x+e)-cos(f*x+e)*cot(f*x+e)+csc(f*x+e))*(1+cos(f*x+e))
/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/(g*sin(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.31

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$$
$$= \left[\frac{\sqrt{2}ag\sqrt{-\frac{1}{ag}}\cos(fx+e)\log\left(-\frac{4\sqrt{2}\left(3\cos(fx+e)^2+(3\cos(fx+e)+4)\sin(fx+e)-\cos(fx+e)-4\right)\sqrt{a\sin(fx+e)+a}\sqrt{g\sin(fx+e)}}{\cos(fx+e)^3+3\cos(fx+e)^2+\left(\cos(fx+e)\right.}\right.}{8} \right.$$

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*a*g*sqrt(-1/(a*g))*cos(f*x + e)*log(-(4*sqrt(2)*(3*cos(f*x +
e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt(a*sin(f*x
 + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-1/(a*g)) - 17*cos(f*x + e)^3 - 3*cos(
f*x + e)^2 - (17*cos(f*x + e)^2 + 14*cos(f*x + e) - 4)*sin(f*x + e) + 18*co
s(f*x + e) + 4)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*co
s(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + 8*sqrt(a*sin(f*x + e)
 + a)*sqrt(g*sin(f*x + e)))/(a*c*f*g*cos(f*x + e)), 1/4*(sqrt(2)*a*g*sqrt(1
/(a*g))*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sq
rt(1/(a*g))*(3*sin(f*x + e) - 1)/(cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)
+ 4*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)))/(a*c*f*g*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx$$
$$= -\frac{\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sin(e+fx)-\sqrt{g\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}\,dx}{c}$$

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] -Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a*sin(e + f*x) + a)*sin(e + f*x) - s
qrt(g*sin(e + f*x))*sqrt(a*sin(e + f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \int -\frac{1}{\sqrt{a\sin(fx+e)+a}(c\sin(fx+e)-c)\sqrt{g\sin(fx+e)}}\,dx$$

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(a*sin(f*x + e) + a)*(c*sin(f*x + e) - c)*sqrt(g*sin(f*x
+ e))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c-c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x
))),x)
```

```
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x
))), x)
```

3.19 $\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$

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Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{f}$$

```
[Out] ln(sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3027, 3556}

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}\log(\sin(e+fx))}{f}$$

```
[In] Int[Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Log[Sin[e + f*x]]*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]])/f
```

Rule 3027

```
Int[(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]])/sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[Sqrt[a + b*Sin[
e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Cos[e + f*x]), Int[Cot[e + f*x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& EqQ[c^2 - d^2, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right)\int\cot(e+fx)\,dx\\ &= \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int\csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{f}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
[Out] ((Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])*Sec[e + f*x]*Sqrt[a*(1 + S
in[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/f
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result	size
default	$\frac{\sec(fx+e)\left(\ln(\csc(fx+e)-\cot(fx+e))-\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)\sqrt{a(1+\sin(fx+e))}\,\sqrt{-c(\sin(fx+e)-1)}}{f}$	68

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/sin(f*x+e),x,method=_RETU
RNVERBOSE)
[Out] 1/f*sec(f*x+e)*(ln(csc(f*x+e)-cot(f*x+e))-ln(2/(1+cos(f*x+e))))*(a*(1+sin(f
*x+e)))^(1/2)*(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.39

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{ac}\log\left(\frac{4\left(256\,ac\cos(fx+e)^5-512\,ac\cos(fx+e)^3+337\,ac\cos(fx+e)+\left(256\,\cos(fx+e)^4-512\,\cos(fx+e)^2+175\right)\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{\cdots}\right.}{\cos(fx+e)^3-\cos(fx+e)}\right.}{2\,f}\right.$$
$$\left. -\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\left(16\,\cos(fx+e)^2-7\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{16\,ac\cos(fx+e)^3-25\,ac\cos(fx+e)}\right)}{f}\right]$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(a*c)*log(4*(256*a*c*cos(f*x + e)^5 - 512*a*c*cos(f*x + e)^3 + 337
*a*c*cos(f*x + e) + (256*cos(f*x + e)^4 - 512*cos(f*x + e)^2 + 175)*sqrt(a*
c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(cos(f*x + e)^3 - co
s(f*x + e)))/f, -sqrt(-a*c)*arctan(sqrt(-a*c)*(16*cos(f*x + e)^2 - 7)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(16*a*c*cos(f*x + e)^3 - 25*a
*c*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2)/sin(f*x+e),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))/sin(e + f*x
), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = \int \frac{\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{\sin(fx+e)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/sin(f*x + e),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\sqrt{a}\sqrt{c}\log\left(\left|2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="giac")
```

```
[Out] -sqrt(a)*sqrt(c)*log(abs(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx = \int \frac{\sqrt{a+a\sin(e+f\,x)}\,\sqrt{c-c\sin(e+f\,x)}}{\sin(e+f\,x)}\,dx$$

```
[In] int(((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/sin(e + f*x),x)
```

```
[Out] int(((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/sin(e + f*x), x
)
```

3.20 $\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

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Mathematica [A] (verified) 149
Maple [A] (verified) 150
Fricas [F] 150
Sympy [F] 150
Maxima [A] (verification not implemented) 151
Giac [A] (verification not implemented) 151
Mupad [F(-1)] 151

Optimal result

Integrand size = 36, antiderivative size = 102

$$\begin{aligned}\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\ &\quad + \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{cf}\end{aligned}$$

```
[Out] -a*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+ln(sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)
/c/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3021, 2816, 2746, 31, 3027, 3556}

$$\begin{aligned}\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}\log(\sin(e+fx))}{cf} \\ &\quad - \frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])) + (Log[Sin[e + f*x]]*Sec[e + f*x]*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]])/(c*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3021

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-d/c, Int[Sqrt[
a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/c, Int[Sqrt[a
 + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Sin[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[b*
c + a*d, 0]
```

Rule 3027

```
Int[(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]])/sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[Sqrt[a + b*Sin[
e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Cos[e + f*x]), Int[Cot[e + f*x], x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& EqQ[c^2 - d^2, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
&\text{integral} \\
&= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{c} + \int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx \\
&= \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right)\int \cot(e+fx)\,dx}{c} \\
&= \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{cf} \\
&\quad - \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{cf}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$
\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(\log(1-\sin(e+fx))-\log(\sin(e+fx)))\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{cf}
$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]], x]

[Out] -(((Log[1 - Sin[e + f*x]] - Log[Sin[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c*f))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{(2\ln(\csc(fx+e)-\cot(fx+e)-1)-\ln(\csc(fx+e)-\cot(fx+e)))\sqrt{a(1+\sin(fx+e))}\,(-\cos(fx+e)+\sin(fx+e)-1)}{f(\cos(fx+e)+\sin(fx+e)+1)\sqrt{-c(\sin(fx+e)-1)}}$	100

[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(2*ln(csc(f*x+e)-cot(f*x+e)-1)-ln(csc(f*x+e)-cot(f*x+e)))*(a*(1+sin(f*x+e)))^(1/2)*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(f*x+e)+sin(f*x+e)+1)/(-c*(sin(f*x+e)-1))^(1/2)

Fricas [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}\,dx$$

[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*cos(f*x + e)^2 + c*sin(f*x + e) - c), x)

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{\sqrt{a(\sin(e+fx)+1)}}{\sqrt{-c(\sin(e+fx)-1)}\sin(e+fx)}\,dx$$

[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(sqrt(-c*(sin(e + f*x) - 1))*sin(e + f*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}}{f}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] (2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(s
in(f*x + e)/(cos(f*x + e) + 1))/sqrt(c))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = 0$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] 0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c - c*sin(e + f*x))^(1/2)),x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c - c*sin(e + f*x))^(1/2)), x
)
```

3.21 $\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

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Optimal result

Integrand size = 36, antiderivative size = 100

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] -c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+ln(sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)
/a/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3021, 2816, 2746, 31, 3027, 3556}

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sec(e+fx)\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}\log(\sin(e+fx))}{af} - \frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])) + (Log[Sin[e + f*x]]*Sec[e + f*x]*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3021

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-d/c, Int[Sqrt[
a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/c, Int[Sqrt[a
 + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Sin[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[b*
c + a*d, 0]
```

Rule 3027

```
Int[(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]])/sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[Sqrt[a + b*Sin[
e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/Cos[e + f*x]), Int[Cot[e + f*x], x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& EqQ[c^2 - d^2, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{a} - \int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= -\frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right)\int \cot(e+fx)\,dx}{a} \\
&= \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{af} \\
&\quad - \frac{(c\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&\quad + \frac{\log(\sin(e+fx))\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}{af}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$
\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(\log(\sin(e+fx)) - \log(1+\sin(e+fx)))\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{af}
$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]], 
x]
```

```
[Out] ((Log[Sin[e + f*x]] - Log[1 + Sin[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sin[e
 + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(2\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln(\csc(fx+e)-\cot(fx+e)))\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)+\sin(fx+e)+1)}{f(-\cos(fx+e)+\sin(fx+e)-1)\sqrt{a(1+\sin(fx+e))}}$	100

```
[In] int((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(2*ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(csc(f*x+e)-cot(f*x+e)))*(-c*(sin(f*x
+e)-1))^(1/2)*(cos(f*x+e)+sin(f*x+e)+1)/(-cos(f*x+e)+sin(f*x+e)-1)/(a*(1+si
n(f*x+e)))^(1/2)
```

Fricas [F]

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*cos(f*x + e
)^2 - a*sin(f*x + e) - a), x)
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(sqrt(a*(sin(e + f*x) + 1))*sin(e + f*
x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{a}}}{f}$$

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] (2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1))/sqrt(a))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\sqrt{c}\left(\frac{\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\log\left(\left|2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right|\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{2f}$$

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*sqrt(c)*(sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
+ 1)/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*log(abs(2*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2 - 1))/(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{c-c\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\sqrt{a+a\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((c - c*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)), x
)
```

3.22 $\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

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Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\log(\tan(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*ln(tan(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3025, 2700, 29}

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*Log[Tan[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3025

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*S
qrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/
(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[1/(Cos[e + f*x]*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cos(e+fx)\int\csc(e+fx)\sec(e+fx)\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\text{Subst}\left(\int\frac{1}{x}\,dx,x,\tan(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\log(\tan(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int\frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(\log(\cos(e+fx))-\log(\sin(e+fx)))\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] -(((Log[Cos[e + f*x]] - Log[Sin[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sin[e +
 f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(42) = 84.

Time = 1.79 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\cos(fx+e)(\ln(\csc(fx+e)-\cot(fx+e))-\ln(\csc(fx+e)-\cot(fx+e)-1)-\ln(-\cot(fx+e)+\csc(fx+e)+1))}{f\sqrt{a(1+\sin(fx+e))}\sqrt{-c(\sin(fx+e)-1)}}$	91

```
[In] int(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*cos(f*x+e)*(ln(csc(f*x+e)-cot(f*x+e))-ln(csc(f*x+e)-cot(f*x+e)-1)-ln(-c
ot(f*x+e)+csc(f*x+e)+1))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.20

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{\sqrt{ac}\log\left(-\frac{4\left(2\,ac\cos(fx+e)^5-2\,ac\cos(fx+e)^3+ac\cos(fx+e)-\sqrt{ac}\left(2\,\cos(fx+e)^2-1\right)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\right)}{\cos(fx+e)^5-\cos(fx+e)^3}\right)}{2\,acf}, \sqrt{-c}\right.$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*sqrt(a*c)*log(-4*(2*a*c*cos(f*x + e)^5 - 2*a*c*cos(f*x + e)^3 + a*c*co
s(f*x + e) - sqrt(a*c)*(2*cos(f*x + e)^2 - 1)*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c))/(cos(f*x + e)^5 - cos(f*x + e)^3))/(a*c*f), sqrt(-a*
c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(2*
a*c*cos(f*x + e)^3 - a*c*cos(f*x + e)))/(a*c*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\sin\left(e+fx\right)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*sin(e +
f*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.13

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(-1)^{4\cos(2fx+2e)}\cosh(4\pi\sin(2fx+2e))\log\left(\frac{16\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)}{ac\left|e^{(2ifx+2ie)}-1\right|^2}\right)-2i(-1)^{4c}}{2\sqrt{a}\sqrt{c}f}$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((-1)^(4*cos(2*f*x + 2*e))*cosh(4*pi*sin(2*f*x + 2*e))*log(16*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)/(a*c*abs(e^(2*I*
f*x + 2*I*e) - 1)^2)) - 2*I*(-1)^(4*cos(2*f*x + 2*e))*arctan2(4*sin(2*f*x +
 2*e)/(sqrt(a)*sqrt(c)*abs(e^(2*I*f*x + 2*I*e) - 1)), 4*(cos(2*f*x + 2*e) +
 1)/(sqrt(a)*sqrt(c)*abs(e^(2*I*f*x + 2*I*e) - 1)))*sinh(4*pi*sin(2*f*x + 2
*e)))/(sqrt(a)*sqrt(c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{1}{\sin(e+fx)\,\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),
 x)
```

3.23 $\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

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Optimal result

Integrand size = 33, antiderivative size = 105

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{c\sqrt{c+d}f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/c/f+2*arctanh
(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)*d^(
1/2)/c/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3013, 2852, 212, 214}

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{cf\sqrt{c+d}} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{cf}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(c*f)
 + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*S
qrt[a + a*Sin[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3013

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*((c
_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c, Int[Sqrt[a + b
*Sin[e + f*x]]/Sin[e + f*x], x], x] - Dist[d/c, Int[Sqrt[a + b*Sin[e + f*x]
]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{c} - \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{c}\\
&= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{(2ad)\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf}\\
&= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} + \frac{2\sqrt{a}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{c\sqrt{c+d}f}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.01 (sec) , antiderivative size = 746, normalized size of antiderivative = 7.10

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = -\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left((4+4i)\sqrt{c+d}\left(\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\right.\right.\right.}{}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((-1/8 + I/8)*((4 + 4*I)*Sqrt[c + d]*(Log[1 + Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] - Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Sqrt[d]*RootSum[-
d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e
)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*S
qrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1
- I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1
^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]
*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/((-I)*d - c*E^(I*e)*#1^2
) & ]*(Cos[e/2] + I*Sin[e/2]) + Sqrt[d]*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 +
 d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[
E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sq
rt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^
((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I
*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E
^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e/2] + I*Sin[e/2])
)*Sqrt[a*(1 + Sin[e + f*x])]])/(c*Sqrt[c + d]*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(d\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{a(c+d)d}}\right)a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a}}\right)a\sqrt{a(c+d)d}\right)}{\sqrt{a}\,c\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	120

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/a^(1/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(d*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)-arctanh((-a*(sin(f*x+e)-1))^(1/2)
/a^(1/2))*a*(a*(c+d)*d)^(1/2))/c/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+
e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(85) = 170$.

Time = 0.58 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.44

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{\frac{ad}{c+d}}\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2\,acd-ad^2-(6\,acd+7\,ad^2)\cos(fx+e)^2+4\left((cd+d^2)\cos(fx+e)^2-c^2-4\,cd-3\,d^2-(c^2+3\,cd+2\,d^2)\cos(f\right.}{d^2\cos(fx+e)^3+(2\,cd+d^2)\cos(fx+}\right.}{}\right.$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*(sqrt(a*d/(c + d))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2
 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2
 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^
2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a
*d/(c + d)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos
(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f
*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f
*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f
*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)))/(c*f),
1/2*(2*sqrt(-a*d/(c + d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
e) - c - 2*d)*sqrt(-a*d/(c + d))/(a*d*cos(f*x + e))) + sqrt(a)*log((a*cos(f
*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin
(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(
f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos
(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x
+ e) - 1)))/(c*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/((c + d*sin(e + f*x))*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sin(f*x + e)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.36

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \frac{\sqrt{2}\left(\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-cd-d^2}c} - \frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}\right.\right.}{c}\right.}{2f}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*sqrt(2)*(2*sqrt(2)*d*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sq
rt(-c*d - d^2))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/(sqrt(-c*d - d^2)*c) -
sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e))/abs(2*sqrt(2
) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/
c)*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx=\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))),x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))), x)
```

3.24 $\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 33, antiderivative size = 165

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/c/f/a^(1/2)+arctanh(1
/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/
2)-2*d^(3/2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e)
)^(1/2))/c/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

= {3019, 2852, 214, 3064, 2728, 212}

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}cf}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(Sqrt[a]*c*f)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x]
)/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*c*(c - d)*Sqrt[c + d]*f
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3019

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[d^2/(c*(b*c - a*d
```

```
)), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] + Dist[1/(c*(
b*c - a*d)), Int[(b*c - a*d - b*d*Sin[e + f*x])/(Sin[e + f*x]*Sqrt[a + b*Si
n[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\csc(e+fx)(ac-ad-ad\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac(c-d)} + \frac{d^2\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{ac(c-d)} \\
&= \frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{ac} - \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} \\
&\quad - \frac{(2d^2)\operatorname{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{c(c-d)f} \\
&= -\frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f} - \frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{cf} \\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} \\
&= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} \\
&\quad - \frac{2d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.37 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.96

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \frac{\left(-2\sqrt{c+d}\left((2+2i)(-1)^{3/4}c\,\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+(c-d)\left(\log\left(1+\cos\left(\right.\right.\right.\right.\right.}{}$$

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((-2*Sqrt[c + d]*((2 + 2*I)*(-1)^(3/4)*c*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1
 + Tan[(e + f*x)/4])] + (c - d)*(Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] - Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) - d^(3/2)*RootSum[c + 4
*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]])
+ Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*
x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/
4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1
^3) & ] + d^(3/2)*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(
d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x
)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^
3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]))/(2*c*(c - d)*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.25

method	result
default	$\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-2d^2\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{acd+a\,d^2}}\right)a^{\frac{5}{2}}+\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2c\sqrt{a(c+d)d}-2\,\text{arc}\right.}{(c-d)c\sqrt{a(c+d)d}\,a^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e}}$

```
[In] int(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-2*d^2*arctanh((a-a*sin(f*x+e))^(
1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*c*(a*(c+d)*d)^(1/2)-2*arctanh((a-a*sin(f*x+e))^(1/
2)/a^(1/2))*a^2*(a*(c+d)*d)^(1/2)*c+2*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2
```

```
))*a^2*(a*(c+d)*d)^(1/2)*d)/(c-d)/c/(a*(c+d)*d)^(1/2)/a^(5/2)/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(136) = 272$.

Time = 1.51 (sec) , antiderivative size = 1044, normalized size of antiderivative = 6.33

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*co
s(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*
c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (
c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c
 + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x +
 e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(
f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*si
n(f*x + e))) + sqrt(2)*sqrt(a)*c*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*
sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x +
 e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - sqrt(a)*(c - d)*log((a*cos(f*x + e)^3
- 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e)
- 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) +
 (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^
3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1))
)/((a*c^2 - a*c*d)*f), -1/2*(2*a*d*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*s
in(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x
 + e))) + sqrt(2)*sqrt(a)*c*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) +
 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) - sqrt(a)*(c - d)*log((a*cos(f*x + e)^3 - 7*a
*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*c
os(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*c
os(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + c
os(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)))/((a
*c^2 - a*c*d)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)\sin\left(e+fx\right)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))*sin(e + f*x)),
x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)\sin\left(fx+e\right)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="maxima")
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(c^2-cd)\sqrt{-cd-d^2}}+\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{c}+\frac{\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{c-d}-\frac{\log(-\cdots}{\cdots}\right)}{2\sqrt{a}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

```
[In] integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*d^2*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)
/sqrt(-c*d - d^2))/((c^2 - c*d)*sqrt(-c*d - d^2)) + sqrt(2)*log(abs(-2*sqrt
(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2
*f*x + 1/2*e)))/c + log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c - d) - log(-
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(c - d))/(sqrt(a)*f*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sin(e+f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),x)
[Out] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)
```

3.25 $\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 149

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = -\frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{df} + \frac{2\sqrt{a}\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{d\sqrt{c+d}f}$$

```
[Out] -2*arctan(cos(f*x+e)*a^(1/2)*g^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^
(1/2))*a^(1/2)*g^(1/2)/d/f+2*arctan(cos(f*x+e)*a^(1/2)*c^(1/2)*g^(1/2)/(c+d
)^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)*c^(1/2)*g^(1/2
)/d/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3007, 2854, 211, 3009}

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{df\sqrt{c+d}} - \frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{df}$$

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[g]*ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[g*Sin[e + f
*x]]*Sqrt[a + a*Sin[e + f*x]])])/(d*f) + (2*Sqrt[a]*Sqrt[c]*Sqrt[g]*ArcTan[
(Sqrt[a]*Sqrt[c]*Sqrt[g]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sin[e + f*x]]*Sq
rt[a + a*Sin[e + f*x]])])/(d*Sqrt[c + d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3007

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]])/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g/d, In
t[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Sin[e + f*x]], x], x] - Dist[c*(g/d), Int
[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^
2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}}\,dx}{d} - \frac{(cg)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{d} \\
&= -\frac{(2ag)\text{Subst}\left(\int \frac{1}{a+gx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{df} \\
&\quad + \frac{(2acg)\text{Subst}\left(\int \frac{1}{ac+ad+cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{df}
\end{aligned}$$

$$= -\frac{2\sqrt{a}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{df} + \frac{2\sqrt{a}\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{d\sqrt{c+d}f}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.44

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$$

$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-\frac{5}{2}i(e+fx)}\left(-1+e^{2i(e+fx)}\right)^{5/2}\left(-2i\sqrt{-1+e^{2i(e+fx)}}+\left(i+\frac{c-d}{\sqrt{-c^2+d^2}}\right)\sqrt{-1+e^{2i(e+fx)}}+\left(i+\frac{-c}{\sqrt{-c^2}}\right.\right.}{}$$

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])/(c + d*Sin[e + f*
x]),x]
```

```
[Out] ((1/2 + I/2)*(-1 + E^((2*I)*(e + f*x)))^(5/2)*((-2*I)*Sqrt[-1 + E^((2*I)*(e
 + f*x))] + (I + (c - d)/Sqrt[-c^2 + d^2])*Sqrt[-1 + E^((2*I)*(e + f*x))] +
 (I + (-c + d)/Sqrt[-c^2 + d^2])*Sqrt[-1 + E^((2*I)*(e + f*x))] + (2*I)*Arc
Tan[Sqrt[-1 + E^((2*I)*(e + f*x))]] + ((I + (-c + d)/Sqrt[-c^2 + d^2])*(Sqr
t[2]*Sqrt[c]*Sqrt[c + I*Sqrt[-c^2 + d^2]]*ArcTan[(d - ((-I)*c + Sqrt[-c^2 +
 d^2])*E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[c]*Sqrt[c + I*Sqrt[-c^2 + d^2]]*Sqrt[
-1 + E^((2*I)*(e + f*x))])] + ((-I)*c + Sqrt[-c^2 + d^2])*ArcTanh[E^(I*(e +
 f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]]))/d + ((I + (c - d)/Sqrt[-c^2 + d^2]
)*(Sqrt[2]*Sqrt[c]*Sqrt[c - I*Sqrt[-c^2 + d^2]]*ArcTan[(d + (I*c + Sqrt[-c^
2 + d^2])*E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[c]*Sqrt[c - I*Sqrt[-c^2 + d^2]]*Sq
rt[-1 + E^((2*I)*(e + f*x))])] - (I*c + Sqrt[-c^2 + d^2])*ArcTanh[E^(I*(e +
 f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]]))/d)*Sqrt[g*Sin[e + f*x]]*Sqrt[a*(1
+ Sin[e + f*x])])/(Sqrt[2]*d*E^(((5*I)/2)*(e + f*x))*(((-I)*(-1 + E^((2*I)*
(e + f*x))))/E^(I*(e + f*x)))^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*Sqrt[Sin[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(117) = 234$.

Time = 3.64 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.28

method	result	size
default	Expression too large to display	935

```
[In] int((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2/f*(g*sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*(2^(1/2)*(-(c-d)*(c+d))
^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2
)*ln(-(csc(f*x+e)-cot(f*x+e)+(csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1)/((csc
(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)-csc(f*x+e)+cot(f*x+e)-1))+4*2^(1/2)*(-(c-
d)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)-d
)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1)+4*2^(1/2)*(-(c-d
)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)-d)
*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)-1)+2^(1/2)*(-(c-d)*(
c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)-d)*c)
^(1/2)*ln(-((csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)-csc(f*x+e)+cot(f*x+e)-1)/
(csc(f*x+e)-cot(f*x+e)+(csc(f*x+e)-cot(f*x+e))^(1/2)*2^(1/2)+1))+4*(-(c-d)*
(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh((csc(f*x+e)-cot(f*x
+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c-4*(-(c-d)*(c+d))^(1/2)*(
((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/((
(-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*c+4*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*ar
ctanh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c
^2-4*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh((csc(f*x+e)-cot(f*x+e))^(1/
2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c*d+4*(((-(c-d)*(c+d))^(1/2)-d)*c)
^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^
(1/2))*c^2-4*(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+
e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*c*d)/(cos(f*x+e)+sin(f*x+e)
+1)/(csc(f*x+e)-cot(f*x+e))^(1/2)/d/(-(c-d)*(c+d))^(1/2)/(((-(c-d)*(c+d))^(
1/2)+d)*c)^(1/2)/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(117) = 234$.

Time = 1.47 (sec) , antiderivative size = 3273, normalized size of antiderivative = 21.97

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a*c*g/(c + d))*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 +
32*a*c*d^3 + a*d^4)*g*cos(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*
d^2 - 4*a*c*d^3 - a*d^4)*g*cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 19
5*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3
*d + 29*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*g*cos(f*x + e)^2 + (289*a*c^4 + 480*
a*c^3*d + 230*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*cos(f*x + e) + 8*((16*c^4 +
 40*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c^4 + 110*c^3*
d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*d^2 + 7*c*d^3
)*cos(f*x + e)^3 - (66*c^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^3 + 2*d^4)*co
s(f*x + e)^2 + (25*c^4 + 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x + e) - (5
1*c^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40*c^3*d + 34*c
^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d + 69*c^2*d^2 +
 18*c*d^3 + d^4)*cos(f*x + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*d^2 + 11*c*d^
3 + d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt(a*sin(f*x +
e) + a)*sqrt(g*sin(f*x + e)) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
 + a*d^4)*g + ((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^
4)*g*cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)
*g*cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 +
 a*d^4)*g*cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c
*d^3)*g*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f*x + e)^4 + c
^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)
^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 + (c^4 + 6*c^2*
d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*x + e)^3 + c^
4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*cos
(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e))) + sqrt(-a*g)*l
og((128*a*g*cos(f*x + e)^5 - 128*a*g*cos(f*x + e)^4 - 416*a*g*cos(f*x + e)^
3 + 128*a*g*cos(f*x + e)^2 + 289*a*g*cos(f*x + e) - 8*(16*cos(f*x + e)^4 -
24*cos(f*x + e)^3 - 66*cos(f*x + e)^2 + (16*cos(f*x + e)^3 + 40*cos(f*x + e
)^2 - 26*cos(f*x + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(-a*g)
*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)) + a*g + (128*a*g*cos(f*x + e
)^4 + 256*a*g*cos(f*x + e)^3 - 160*a*g*cos(f*x + e)^2 - 288*a*g*cos(f*x + e
) + a*g)*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/(d*f), -1/4*(2*s
qrt(a*c*g/(c + d))*arctan(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2
 - 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*sin(f*x + e))*sqrt(a*c*g/(c + d))*sqrt(a
*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(a*c^2*g*cos(f*x + e)*sin(f*x + e)
+ (2*a*c^2 + a*c*d)*g*cos(f*x + e)^3 - (2*a*c^2 + a*c*d)*g*cos(f*x + e))) -
 sqrt(-a*g)*log((128*a*g*cos(f*x + e)^5 - 128*a*g*cos(f*x + e)^4 - 416*a*g*
cos(f*x + e)^3 + 128*a*g*cos(f*x + e)^2 + 289*a*g*cos(f*x + e) - 8*(16*cos(
f*x + e)^4 - 24*cos(f*x + e)^3 - 66*cos(f*x + e)^2 + (16*cos(f*x + e)^3 + 4
0*cos(f*x + e)^2 - 26*cos(f*x + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 5
1)*sqrt(-a*g)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)) + a*g + (128*a*
g*cos(f*x + e)^4 + 256*a*g*cos(f*x + e)^3 - 160*a*g*cos(f*x + e)^2 - 288*a*
g*cos(f*x + e) + a*g)*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/(d*
f), 1/4*(2*sqrt(a*g)*arctan(1/4*sqrt(a*g)*(8*cos(f*x + e)^2 + 8*sin(f*x + e
```

```
) - 9)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(2*a*g*cos(f*x + e)^3
+ a*g*cos(f*x + e)*sin(f*x + e) - 2*a*g*cos(f*x + e))) + sqrt(-a*c*g/(c + d
))*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*co
s(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)
*g*cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d^3
 + a*d^4)*g*cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*a*
c*d^3 - a*d^4)*g*cos(f*x + e)^2 + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*d^2
+ 32*a*c*d^3 + a*d^4)*g*cos(f*x + e) + 8*((16*c^4 + 40*c^3*d + 34*c^2*d^2 +
 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^
3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x + e)^3 - (66*c
^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (25*c^4 +
 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x + e) - (51*c^4 + 110*c^3*d + 76*c
^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)
*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d + 69*c^2*d^2 + 18*c*d^3 + d^4)*cos(f*x
 + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x +
 e)) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g + ((128*a*c^
4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*cos(f*x + e)^4 + 4*
(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*cos(f*x + e)^3 - 2*(8
0*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 + a*d^4)*g*cos(f*x + e)^2
 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^3)*g*cos(f*x + e) + (
a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*g)*sin(f*x + e))/(d^4*
cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2
 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*
d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 + (c^4 + 6*c^2*d^2 + d^4)*cos(f*x + e)
+ (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2
+ 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*cos(f*x + e)^2 + 4*(c^3*d +
 c*d^3)*cos(f*x + e))*sin(f*x + e))))/(d*f), -1/2*(sqrt(a*c*g/(c + d))*arct
an(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 - 8*c*d - d^2 + 2*(4*c
^2 + 3*c*d)*sin(f*x + e))*sqrt(a*c*g/(c + d))*sqrt(a*sin(f*x + e) + a)*sqrt
(g*sin(f*x + e))/(a*c^2*g*cos(f*x + e)*sin(f*x + e) + (2*a*c^2 + a*c*d)*g*c
os(f*x + e)^3 - (2*a*c^2 + a*c*d)*g*cos(f*x + e))) - sqrt(a*g)*arctan(1/4*s
qrt(a*g)*(8*cos(f*x + e)^2 + 8*sin(f*x + e) - 9)*sqrt(a*sin(f*x + e) + a)*s
qrt(g*sin(f*x + e))/(2*a*g*cos(f*x + e)^3 + a*g*cos(f*x + e)*sin(f*x + e) -
 2*a*g*cos(f*x + e))))/(d*f)]
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{g\sin\left(e+fx\right)}}{c+d\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((g*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(g*sin(e + f*x))/(c + d*sin(e + f*x
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx=\int \frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{g\sin\left(fx+e\right)}}{d\sin\left(fx+e\right)+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(d*sin(f*x + e) + c
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx=\int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)
),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)
), x)
```

3.26 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))} dx$

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Optimal result

Integrand size = 39, antiderivative size = 83

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))} dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{c}\sqrt{c+d}f\sqrt{g}}$$

```
[Out] -2*arctan(cos(f*x+e)*a^(1/2)*c^(1/2)*g^(1/2)/(c+d)^(1/2)/(g*sin(f*x+e))^(1/
2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f/c^(1/2)/(c+d)^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {3009, 211}

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))} dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{c}f\sqrt{g}\sqrt{c+d}}$$

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[c]*Sqrt[c + d]*f*Sqrt[g]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2a)\text{Subst}\left(\int \frac{1}{ac+ad+cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{c}\sqrt{c+d}f\sqrt{g}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 436, normalized size of antiderivative = 5.25

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)g\left(\sqrt{c+i\sqrt{-c^2+d^2}}\left(ic-id+\sqrt{-c^2+d^2}\right)\arctan\left(\frac{d-\left(-ic+\sqrt{-c^2+d^2}\right)(\cos(e+fx)+i\sin(e+fx))}{\sqrt{2}\sqrt{c}\sqrt{c+i\sqrt{-c^2+d^2}}\sqrt{-1+\cos(2(e+fx))+i\sin(2(e+fx))}}\right.\right.}{}$$

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] ((1/4 + I/4)*g*(Sqrt[c + I*Sqrt[-c^2 + d^2]]*(I*c - I*d + Sqrt[-c^2 + d^2])
*ArcTan[(d - ((-I)*c + Sqrt[-c^2 + d^2])*(Cos[e + f*x] + I*Sin[e + f*x]))/(
Sqrt[2]*Sqrt[c]*Sqrt[c + I*Sqrt[-c^2 + d^2]]*Sqrt[-1 + Cos[2*(e + f*x)] + I
*Sin[2*(e + f*x)]])] + Sqrt[c - I*Sqrt[-c^2 + d^2]]*((-I)*c + I*d + Sqrt[-c
^2 + d^2])*ArcTan[(d + (I*c + Sqrt[-c^2 + d^2])*(Cos[e + f*x] + I*Sin[e + f
*x]))/(Sqrt[2]*Sqrt[c]*Sqrt[c - I*Sqrt[-c^2 + d^2]]*Sqrt[-1 + Cos[2*(e + f*
x)] + I*Sin[2*(e + f*x)]])])*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[(3*(e + f*x))/
2] - I*Sin[(3*(e + f*x))/2])*(-1 + Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])^(
3/2))/(Sqrt[2]*Sqrt[c]*d*Sqrt[-c^2 + d^2]*f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(g*Sin[e + f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(63) = 126$.

Time = 3.17 (sec) , antiderivative size = 505, normalized size of antiderivative = 6.08

method	result
default	$-\frac{2\sqrt{\csc(fx+e)-\cot(fx+e)}\,\sqrt{a(1+\sin(fx+e))}\left(\sqrt{-(c-d)(c+d)}\,\sqrt{\left(\sqrt{-(c-d)(c+d)}+d\right)c}\,\operatorname{arctanh}\left(\frac{\sqrt{\csc(fx+e)-\cot(fx+e)}\,c}{\sqrt{\left(\sqrt{-(c-d)(c+d)}-d\right)c}}\right)+\mathrm{arc}}{}$

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2/f*(csc(f*x+e)-cot(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*((-(c-d)*(c+d))
^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh((csc(f*x+e)-cot(f*x+e))^(
1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))+arctanh((csc(f*x+e)-cot(f*x+e))^
(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1
/2)*c-arctanh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^
(1/2))*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*d-(-(c-d)*(c+d))^(1/2)*(((-(c-d)*
(c+d))^(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(
c+d))^(1/2)+d)*c)^(1/2))+arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(
c+d))^(1/2)+d)*c)^(1/2))*(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*c-arctan((csc(f
*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*(((-(c-d)*(c+
d))^(1/2)-d)*c)^(1/2)*d)*(1+cos(f*x+e))/(cos(f*x+e)+sin(f*x+e)+1)/(g*sin(f*
x+e))^(1/2)/(-(c-d)*(c+d))^(1/2)/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)/(((-(c-
d)*(c+d))^(1/2)+d)*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(63) = 126$.

Time = 0.89 (sec) , antiderivative size = 1303, normalized size of antiderivative = 15.70

$$\int \frac{\sqrt{a + a\sin(e + fx)}}{\sqrt{g\sin(e + fx)}(c + d\sin(e + fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*sqrt(-a/((c^2 + c*d)*g))*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2
 + 32*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4
*a*c*d^3 + a*d^4 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*
d^4)*cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d
^3 + a*d^4)*cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*a*
c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*(51*c^5 + 110*c^4*d + 76*c^3*d^2 + 18*c^2
```

```
*d^3 + c*d^4 + (16*c^5 + 40*c^4*d + 34*c^3*d^2 + 11*c^2*d^3 + c*d^4)*cos(f*
x + e)^4 - (24*c^5 + 52*c^4*d + 35*c^3*d^2 + 7*c^2*d^3)*cos(f*x + e)^3 - (6
6*c^5 + 149*c^4*d + 110*c^3*d^2 + 29*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (2
5*c^5 + 53*c^4*d + 35*c^3*d^2 + 7*c^2*d^3)*cos(f*x + e) - (51*c^5 + 110*c^4
*d + 76*c^3*d^2 + 18*c^2*d^3 + c*d^4 - (16*c^5 + 40*c^4*d + 34*c^3*d^2 + 11
*c^2*d^3 + c*d^4)*cos(f*x + e)^3 - (40*c^5 + 92*c^4*d + 69*c^3*d^2 + 18*c^2
*d^3 + c*d^4)*cos(f*x + e)^2 + (26*c^5 + 57*c^4*d + 41*c^3*d^2 + 11*c^2*d^3
 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f
*x + e))*sqrt(-a/((c^2 + c*d)*g)) + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*d^
2 + 32*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4
*a*c*d^3 + a*d^4 + (128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 +
a*d^4)*cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d^
3)*cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 18*a*c*d^3 +
 a*d^4)*cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*d^2 + 7*a*c*d
^3)*cos(f*x + e))*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f
*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)
*cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 +
(c^4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*
x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d
^3 + d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e)))/f
, 1/2*sqrt(a/((c^2 + c*d)*g))*arctan(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e
)^2 - 9*c^2 - 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a)*sqrt(g*sin(f*x + e))*sqrt(a/((c^2 + c*d)*g))/((2*a*c + a*d)*cos(f
*x + e)^3 + a*c*cos(f*x + e)*sin(f*x + e) - (2*a*c + a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{g\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(sqrt(g*sin(e + f*x))*(c + d*sin(e + f*
x))), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
), x)
```

3.27 $\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 166

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{\sqrt{2}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

```
[Out] arctan(1/2*cos(f*x+e)*a^(1/2)*g^(1/2)*2^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin
(f*x+e))^(1/2))*2^(1/2)*g^(1/2)/(c-d)/f/a^(1/2)-2*arctan(cos(f*x+e)*a^(1/2)
*c^(1/2)*g^(1/2)/(c+d)^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2))*c
^(1/2)*g^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3015, 2861, 211, 3009}

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{\sqrt{2}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)} - \frac{2\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

```
[In] Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (Sqrt[2]*Sqrt[g]*ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[2]*Sqrt[g*Sin[
e + f*x]]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*f) - (2*Sqrt[c]*Sqrt
[g]*ArcTan[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sin[e
 + f*x]]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3015

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-a)*(g
/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x
] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*
x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{g\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} + \frac{(cg)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{a(c-d)}$$

$$= \frac{(2ag)\text{Subst}\left(\int \frac{1}{2a^2+agx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} - \frac{(2cg)\text{Subst}\left(\int \frac{1}{ac+ad+cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f}$$

$$= \frac{\sqrt{2}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{c}\sqrt{g}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx =$$
$$-\frac{\left(2\sqrt{c}\sqrt{-c^2+d^2}\arctan\left(\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\right)-\sqrt{d-\sqrt{-c^2+d^2}}\left(-c+d+\sqrt{-c^2+d^2}\right)\arctan\left(\frac{\sqrt{c}\sqrt{}}{\sqrt{d}}\right.\right.}{\sqrt{c}(c-d)\sqrt{-c^2+d^2}f}$$

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] -(((2*Sqrt[c]*Sqrt[-c^2 + d^2]*ArcTan[Sqrt[Tan[(e + f*x)/2]]] - Sqrt[d - Sq
rt[-c^2 + d^2]]*(-c + d + Sqrt[-c^2 + d^2])*ArcTan[(Sqrt[c]*Sqrt[Tan[(e + f
*x)/2]])/Sqrt[d - Sqrt[-c^2 + d^2]]] - (c - d + Sqrt[-c^2 + d^2])*Sqrt[d +
Sqrt[-c^2 + d^2]]*ArcTan[(Sqrt[c]*Sqrt[Tan[(e + f*x)/2]])/Sqrt[d + Sqrt[-c^
2 + d^2]]])*Sqrt[g*Sin[e + f*x]]*(1 + Tan[(e + f*x)/2]))/(Sqrt[c]*(c - d)*S
qrt[-c^2 + d^2]*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[Tan[(e + f*x)/2]]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(131) = 262.

Time = 3.14 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.55

method	result
default	$\frac{\sqrt{g\sin(fx+e)}\left(\sqrt{-(c-d)(c+d)}\sqrt{\left(\sqrt{-(c-d)(c+d)}-d\right)c}\arctan\left(\frac{\sqrt{\csc(fx+e)-\cot(fx+e)}\,c}{\sqrt{\left(\sqrt{-(c-d)(c+d)}+d\right)c}}\right)c-\sqrt{\left(\sqrt{-(c-d)(c+d)}-d\right)c}\arctan\left(\right.\right.}{}$

```
[In] int((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(g*sin(f*x+e))^(1/2)*((-(c-d)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)-d)*c)
^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^
(1/2))*c-(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^
(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*c^2+(((-(c-d)*(c+d))^(1/2)-d)*c
)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)
^(1/2))*c*d-(-(c-d)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh
```

```
((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c-(((-
(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-
(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c^2+(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arct
anh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c*d
-2*arctan((csc(f*x+e)-cot(f*x+e))^(1/2))*(-(c-d)*(c+d))^(1/2)*(((-(c-d)*(c+
d))^(1/2)-d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*(cos(f*x+e)+sin(f
*x+e)+1)/(1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(csc(f*x+e)-cot(f*x+e))^(1
/2)/(c-d)/(-(c-d)*(c+d))^(1/2)/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)/(((-(c-d)
*(c+d))^(1/2)+d)*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.33 (sec) , antiderivative size = 3048, normalized size of antiderivative = 18.36

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*sqrt(-g/a)*log((17*g*cos(f*x + e)^3 - 4*sqrt(2)*(3*cos(f*x +
 e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt(a*sin(f*
x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e)^2 - 18*g*cos
(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*sin(f*x + e) -
4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e)
- 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + sqrt(-c*g/(a*c + a*d))*log(((128
*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^5 - (128*c^
4 + 192*c^3*d + 64*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e)^4 - 2*(208*c^4 +
 368*c^3*d + 195*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^3 + 2*(64*c^4 + 9
4*c^3*d + 29*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e)^2 + (289*c^4 + 480*c^3
*d + 230*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e) + 8*((16*c^4 + 40*c^3*d +
 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c^4 + 110*c^3*d + 76*c^2*
d^2 + 18*c*d^3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x +
 e)^3 - (66*c^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^3 + 2*d^4)*cos(f*x + e)^
2 + (25*c^4 + 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(f*x + e) - (51*c^4 + 110
*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11
*c*d^3 + d^4)*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d + 69*c^2*d^2 + 18*c*d^3 +
 d^4)*cos(f*x + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*d^2 + 11*c*d^3 + d^4)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqr
t(-c*g/(a*c + a*d)) + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*g + ((128
*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e)^4 + 4*(64*c
^4 + 112*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e)^3 - 2*(80*c^4 + 144*c
^3*d + 83*c^2*d^2 + 18*c*d^3 + d^4)*g*cos(f*x + e)^2 - 4*(72*c^4 + 119*c^3*
```

```
d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e) + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f*x
+ e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*co
s(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 + (c^
4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*x +
 e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3
+ d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e))))/((c
 - d)*f), -1/4*(sqrt(2)*sqrt(-g/a)*log((17*g*cos(f*x + e)^3 - 4*sqrt(2)*(3*
cos(f*x + e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt
(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-g/a) + 3*g*cos(f*x + e)^2 -
 18*g*cos(f*x + e) + (17*g*cos(f*x + e)^2 + 14*g*cos(f*x + e) - 4*g)*sin(f*
x + e) - 4*g)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(
f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) - 2*sqrt(c*g/(a*c + a*d))
*arctan(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 - 8*c*d - d^2 + 2
*(4*c^2 + 3*c*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)
)*sqrt(c*g/(a*c + a*d))/((2*c^2 + c*d)*g*cos(f*x + e)^3 + c^2*g*cos(f*x + e
)*sin(f*x + e) - (2*c^2 + c*d)*g*cos(f*x + e))))/((c - d)*f), -1/4*(2*sqrt(
2)*sqrt(g/a)*arctan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e
))*sqrt(g/a)*(3*sin(f*x + e) - 1)/(g*cos(f*x + e)*sin(f*x + e))) + sqrt(-c*
g/(a*c + a*d))*log(((128*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*
cos(f*x + e)^5 - (128*c^4 + 192*c^3*d + 64*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f
*x + e)^4 - 2*(208*c^4 + 368*c^3*d + 195*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*
x + e)^3 + 2*(64*c^4 + 94*c^3*d + 29*c^2*d^2 - 4*c*d^3 - d^4)*g*cos(f*x + e
)^2 + (289*c^4 + 480*c^3*d + 230*c^2*d^2 + 32*c*d^3 + d^4)*g*cos(f*x + e) +
 8*((16*c^4 + 40*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^4 + 51*c
^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (24*c^4 + 52*c^3*d + 35*c^2*
d^2 + 7*c*d^3)*cos(f*x + e)^3 - (66*c^4 + 149*c^3*d + 110*c^2*d^2 + 29*c*d^
3 + 2*d^4)*cos(f*x + e)^2 + (25*c^4 + 53*c^3*d + 35*c^2*d^2 + 7*c*d^3)*cos(
f*x + e) - (51*c^4 + 110*c^3*d + 76*c^2*d^2 + 18*c*d^3 + d^4 - (16*c^4 + 40
*c^3*d + 34*c^2*d^2 + 11*c*d^3 + d^4)*cos(f*x + e)^3 - (40*c^4 + 92*c^3*d +
 69*c^2*d^2 + 18*c*d^3 + d^4)*cos(f*x + e)^2 + (26*c^4 + 57*c^3*d + 41*c^2*
d^2 + 11*c*d^3 + d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*
sqrt(g*sin(f*x + e))*sqrt(-c*g/(a*c + a*d)) + (c^4 + 4*c^3*d + 6*c^2*d^2 +
4*c*d^3 + d^4)*g + ((128*c^4 + 256*c^3*d + 160*c^2*d^2 + 32*c*d^3 + d^4)*g*
cos(f*x + e)^4 + 4*(64*c^4 + 112*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x +
e)^3 - 2*(80*c^4 + 144*c^3*d + 83*c^2*d^2 + 18*c*d^3 + d^4)*g*cos(f*x + e)^
2 - 4*(72*c^4 + 119*c^3*d + 56*c^2*d^2 + 7*c*d^3)*g*cos(f*x + e) + (c^4 + 4
*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 +
(4*c*d^3 + d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
- 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d
^4)*cos(f*x + e)^2 + (c^4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x +
e)^4 - 4*c*d^3*cos(f*x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 -
 2*(3*c^2*d^2 + 2*c*d^3 + d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x +
 e))*sin(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)*sqrt(g/a)*arctan(1/4*sqrt(2
)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(g/a)*(3*sin(f*x + e) -
```

```
1)/(g*cos(f*x + e)*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*arctan(1/4*((8*c
^2 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 - 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*
sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(c*g/(a*c +
a*d))/((2*c^2 + c*d)*g*cos(f*x + e)^3 + c^2*g*cos(f*x + e)*sin(f*x + e) -
(2*c^2 + c*d)*g*cos(f*x + e))))/((c - d)*f)]
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(e+fx)}}}{\sqrt{a(\sin{(e+fx)}+1)}\,(c+d\sin{(e+fx)})}\,dx$$

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(g*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*
x))), x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(fx+e)}}}{\sqrt{a\sin{(fx+e)}+a}(d\sin{(fx+e)}+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$=\int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
),x)
[Out] int((g*sin(e + f*x))^(1/2)/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
), x)
```

3.28 $\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 168

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f\sqrt{g}} + \frac{2d\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}(c-d)\sqrt{c+d}f\sqrt{g}}$$

```
[Out] -arctan(1/2*cos(f*x+e)*a^(1/2)*g^(1/2)*2^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*si
n(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)/g^(1/2)+2*d*arctan(cos(f*x+e)*a^(1
/2)*c^(1/2)*g^(1/2)/(c+d)^(1/2)/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/(c-d)/f/a^(1/2)/c^(1/2)/(c+d)^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3017, 2861, 211, 3009}

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{2d\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f\sqrt{g}(c-d)\sqrt{c+d}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{g\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{g}(c-d)}$$

```
[In] Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[g]*Cos[e + f*x])/(Sqrt[2]*Sqrt[g*Sin[e + f*
x]]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)*f*Sqrt[g])) + (2*d*ArcTan[
```

```
(Sqrt[a]*Sqrt[c]*Sqrt[g]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sin[e + f*x]]*Sq
rt[a + a*Sin[e + f*x]])])/(Sqrt[a]*Sqrt[c]*(c - d)*Sqrt[c + d]*f*Sqrt[g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3009

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-2*(b/f
), Subst[Int[1/(b*c + a*d + c*g*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[g*Sin[e
+ f*x]]*Sqrt[a + b*Sin[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3017

```
Int[1/(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*
c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - D
ist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c +
d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*
d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{a(c-d)} \\
&= -\frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2+agx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} \\
&\quad + \frac{(2d)\text{Subst}\left(\int \frac{1}{ac+ad+cgx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} \\
&= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{g}\cos(e+fx)}{\sqrt{2}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f\sqrt{g}} + \frac{2d\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\cos(e+fx)}{\sqrt{c+d}\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}(c-d)\sqrt{c+d}f\sqrt{g}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2\left(-2\arctan\left(\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}\right)+\frac{d\left(1+\frac{c-d}{\sqrt{-c^2+d^2}}\right)\arctan\left(\frac{\sqrt{c}\sqrt{\tan\left(\frac{1}{2}(e+fx)\right)}}{\sqrt{d-\sqrt{-c^2+d^2}}}\right)}{\sqrt{c}\sqrt{d-\sqrt{-c^2+d^2}}}+\frac{d\left(-c+d+\sqrt{-c^2+d^2}\right)\arctan\left(\frac{\sqrt{c}\sqrt{\tan\cdots}}{\sqrt{d+\cdots}}\right)}{\sqrt{c}\sqrt{-c^2+d^2}\sqrt{d+\sqrt{-c^2+d\cdots}}}\cdots}{(c-d)f\sqrt{g\sin(e+fx)}\sqrt{a(1+\cdots}}$$

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f
*x])),x]
```

```
[Out] (-2*(-2*ArcTan[Sqrt[Tan[(e + f*x)/2]]] + (d*(1 + (c - d)/Sqrt[-c^2 + d^2])*
ArcTan[(Sqrt[c]*Sqrt[Tan[(e + f*x)/2]])/Sqrt[d - Sqrt[-c^2 + d^2]]])/(Sqrt[
c]*Sqrt[d - Sqrt[-c^2 + d^2]]) + (d*(-c + d + Sqrt[-c^2 + d^2])*ArcTan[(Sqr
t[c]*Sqrt[Tan[(e + f*x)/2]])/Sqrt[d + Sqrt[-c^2 + d^2]]])/(Sqrt[c]*Sqrt[-c^
2 + d^2]*Sqrt[d + Sqrt[-c^2 + d^2]]))*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[Tan[(e + f*x)/2]])/((c - d)*f*Sqrt[g*Sin[e + f*x]]*S
qrt[a*(1 + Sin[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(133) = 266.

Time = 3.49 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.45

method	result
default	$-\frac{\sqrt{\csc(fx+e)-\cot(fx+e)}\left(\sqrt{-(c-d)(c+d)}\sqrt{\left(\sqrt{-(c-d)(c+d)}-d\right)c}\arctan\left(\frac{\sqrt{\csc(fx+e)-\cot(fx+e)}\,c}{\sqrt{\left(\sqrt{-(c-d)(c+d)}+d\right)c}}\right)d-\sqrt{\left(\sqrt{-(c-d)(c+d)}-d\right)\cdots}\cdots\right.}{\cdots}$

```
[In] int(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/f*(csc(f*x+e)-cot(f*x+e))^(1/2)*((-(c-d)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(
1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1
/2)+d)*c)^(1/2))*d-(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-co
t(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*c*d+(((-(c-d)*(c+d))^
(1/2)-d)*c)^(1/2)*arctan((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(
1/2)+d)*c)^(1/2))*d^2-(-(c-d)*(c+d))^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/
2)*arctanh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/
```

```
2))*d-(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2)*arctanh((csc(f*x+e)-cot(f*x+e))^(1
/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2))*c*d+(((-(c-d)*(c+d))^(1/2)+d)*c)^
(1/2)*arctanh((csc(f*x+e)-cot(f*x+e))^(1/2)*c/(((-(c-d)*(c+d))^(1/2)-d)*c)^
(1/2))*d^2-2*arctan((csc(f*x+e)-cot(f*x+e))^(1/2))*(-(c-d)*(c+d))^(1/2)*(((
-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)*(((-(c-d)*(c+d))^(1/2)+d)*c)^(1/2))*(cos(f*
x+e)+sin(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(g*sin(f*x+e))^(1/2)/(c-d)/(-(c
-d)*(c+d))^(1/2)/(((-(c-d)*(c+d))^(1/2)-d)*c)^(1/2)/(((-(c-d)*(c+d))^(1/2)+
d)*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(133) = 266$.

Time = 1.58 (sec) , antiderivative size = 3175, normalized size of antiderivative = 18.90

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(-1/(a*g))*log((4*sqrt(2)*(3*cos(f*x +
 e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x + e) - 4)*sqrt(a*sin(f*
x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-1/(a*g)) + 17*cos(f*x + e)^3 + 3*cos
(f*x + e)^2 + (17*cos(f*x + e)^2 + 14*cos(f*x + e) - 4)*sin(f*x + e) - 18*c
os(f*x + e) - 4)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*c
os(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) - sqrt(-(a*c^2 + a*c*d
)*g)*d*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*
g*cos(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*
d^4)*g*cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c
*d^3 + a*d^4)*g*cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 -
4*a*c*d^3 - a*d^4)*g*cos(f*x + e)^2 + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*
d^2 + 32*a*c*d^3 + a*d^4)*g*cos(f*x + e) + 8*((16*c^3 + 24*c^2*d + 10*c*d^2
 + d^3)*cos(f*x + e)^4 - (24*c^3 + 28*c^2*d + 7*c*d^2)*cos(f*x + e)^3 + 51*
c^3 + 59*c^2*d + 17*c*d^2 + d^3 - (66*c^3 + 83*c^2*d + 27*c*d^2 + 2*d^3)*co
s(f*x + e)^2 + (25*c^3 + 28*c^2*d + 7*c*d^2)*cos(f*x + e) + ((16*c^3 + 24*c
^2*d + 10*c*d^2 + d^3)*cos(f*x + e)^3 - 51*c^3 - 59*c^2*d - 17*c*d^2 - d^3
+ (40*c^3 + 52*c^2*d + 17*c*d^2 + d^3)*cos(f*x + e)^2 - (26*c^3 + 31*c^2*d
+ 10*c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-(a*c^2 + a*c*d)*g)*sqrt
(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^
2 + 4*a*c*d^3 + a*d^4)*g + ((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a
*c*d^3 + a*d^4)*g*cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2
 + 7*a*c*d^3)*g*cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 +
 18*a*c*d^3 + a*d^4)*g*cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^
2*d^2 + 7*a*c*d^3)*g*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*
```

```
c*d^3 + a*d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f
*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)
*cos(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 +
(c^4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*
x + e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d
^3 + d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e))))/
((a*c^3 - a*c*d^2)*f*g), 1/4*(2*sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(1/(a*g))*arc
tan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(1/(a*g))
*(3*sin(f*x + e) - 1)/(cos(f*x + e)*sin(f*x + e))) + sqrt(-(a*c^2 + a*c*d)*
g)*d*log(((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c*d^3 + a*d^4)*g*
cos(f*x + e)^5 - (128*a*c^4 + 192*a*c^3*d + 64*a*c^2*d^2 - 4*a*c*d^3 - a*d^
4)*g*cos(f*x + e)^4 - 2*(208*a*c^4 + 368*a*c^3*d + 195*a*c^2*d^2 + 32*a*c*d
^3 + a*d^4)*g*cos(f*x + e)^3 + 2*(64*a*c^4 + 94*a*c^3*d + 29*a*c^2*d^2 - 4*
a*c*d^3 - a*d^4)*g*cos(f*x + e)^2 + (289*a*c^4 + 480*a*c^3*d + 230*a*c^2*d^
2 + 32*a*c*d^3 + a*d^4)*g*cos(f*x + e) + 8*((16*c^3 + 24*c^2*d + 10*c*d^2 +
 d^3)*cos(f*x + e)^4 - (24*c^3 + 28*c^2*d + 7*c*d^2)*cos(f*x + e)^3 + 51*c^
3 + 59*c^2*d + 17*c*d^2 + d^3 - (66*c^3 + 83*c^2*d + 27*c*d^2 + 2*d^3)*cos(
f*x + e)^2 + (25*c^3 + 28*c^2*d + 7*c*d^2)*cos(f*x + e) + ((16*c^3 + 24*c^2
*d + 10*c*d^2 + d^3)*cos(f*x + e)^3 - 51*c^3 - 59*c^2*d - 17*c*d^2 - d^3 +
(40*c^3 + 52*c^2*d + 17*c*d^2 + d^3)*cos(f*x + e)^2 - (26*c^3 + 31*c^2*d +
10*c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-(a*c^2 + a*c*d)*g)*sqrt(a
*sin(f*x + e) + a)*sqrt(g*sin(f*x + e)) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2
+ 4*a*c*d^3 + a*d^4)*g + ((128*a*c^4 + 256*a*c^3*d + 160*a*c^2*d^2 + 32*a*c
*d^3 + a*d^4)*g*cos(f*x + e)^4 + 4*(64*a*c^4 + 112*a*c^3*d + 56*a*c^2*d^2 +
 7*a*c*d^3)*g*cos(f*x + e)^3 - 2*(80*a*c^4 + 144*a*c^3*d + 83*a*c^2*d^2 + 1
8*a*c*d^3 + a*d^4)*g*cos(f*x + e)^2 - 4*(72*a*c^4 + 119*a*c^3*d + 56*a*c^2*
d^2 + 7*a*c*d^3)*g*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*
d^3 + a*d^4)*g)*sin(f*x + e))/(d^4*cos(f*x + e)^5 + (4*c*d^3 + d^4)*cos(f*x
 + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + d^4)*c
os(f*x + e)^3 - 2*(2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e)^2 + (c
^4 + 6*c^2*d^2 + d^4)*cos(f*x + e) + (d^4*cos(f*x + e)^4 - 4*c*d^3*cos(f*x
+ e)^3 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(3*c^2*d^2 + 2*c*d^3
 + d^4)*cos(f*x + e)^2 + 4*(c^3*d + c*d^3)*cos(f*x + e))*sin(f*x + e))))/((
a*c^3 - a*c*d^2)*f*g), -1/4*(sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(-1/(a*g))*log((
4*sqrt(2)*(3*cos(f*x + e)^2 + (3*cos(f*x + e) + 4)*sin(f*x + e) - cos(f*x +
 e) - 4)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(-1/(a*g)) + 17*
cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (17*cos(f*x + e)^2 + 14*cos(f*x + e) -
4)*sin(f*x + e) - 18*cos(f*x + e) - 4)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
 (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4))
+ 2*sqrt((a*c^2 + a*c*d)*g)*d*arctan(1/4*((8*c^2 + 8*c*d + d^2)*cos(f*x + e
)^2 - 9*c^2 - 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*sin(f*x + e))*sqrt((a*c^2 + a
*c*d)*g)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/((2*a*c^3 + 3*a*c^2*
d + a*c*d^2)*g*cos(f*x + e)^3 + (a*c^3 + a*c^2*d)*g*cos(f*x + e)*sin(f*x +
e) - (2*a*c^3 + 3*a*c^2*d + a*c*d^2)*g*cos(f*x + e))))/((a*c^3 - a*c*d^2)*f
*g), 1/2*(sqrt(2)*(a*c^2 + a*c*d)*g*sqrt(1/(a*g))*arctan(1/4*sqrt(2)*sqrt(a
```

```
*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))*sqrt(1/(a*g))*(3*sin(f*x + e) - 1)/
(cos(f*x + e)*sin(f*x + e))) - sqrt((a*c^2 + a*c*d)*g)*d*arctan(1/4*((8*c^2
 + 8*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 - 8*c*d - d^2 + 2*(4*c^2 + 3*c*d)*si
n(f*x + e))*sqrt((a*c^2 + a*c*d)*g)*sqrt(a*sin(f*x + e) + a)*sqrt(g*sin(f*x
 + e))/((2*a*c^3 + 3*a*c^2*d + a*c*d^2)*g*cos(f*x + e)^3 + (a*c^3 + a*c^2*d
)*g*cos(f*x + e)*sin(f*x + e) - (2*a*c^3 + 3*a*c^2*d + a*c*d^2)*g*cos(f*x +
 e))))/((a*c^3 - a*c*d^2)*f*g)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{g\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(g*sin(e + f*x))*(c + d*sin(e +
f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sqrt(g*sin(f*x +
 e))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Valueindex.cc index_m operator + Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))),x)
```

```
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))), x)
```

3.29 $\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx$

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Rubi [A] (verified) . . . 202
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Optimal result

Integrand size = 33, antiderivative size = 238

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{cf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{(a-b)\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c+c\sin(e+fx))}$$

```
[Out] cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f/(c+c*sin(f*x+e))-(sin(1/2*e+1/4*Pi+1/2*
f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/c/f/((a+b*sin(f*x+e))/(a+b
))^(1/2)+(a-b)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x
)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*
x+e))/(a+b))^(1/2)/c/f/(a+b*sin(f*x+e))^(1/2)-2*a*(sin(1/2*e+1/4*Pi+1/2*f*x
)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/c/f/(a+b*sin(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3014, 2886, 2884, 2847, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx$$
$$=\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c\sin(e+fx)+c)}-\frac{(a-b)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}}$$
$$+\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{cf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$
$$+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*x]),x]
```

```
[Out] (EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(c*
f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - ((a - b)*EllipticF[(e - Pi/2 + f*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(c*f*Sqrt[a + b*Sin[
e + f*x]]) + (2*a*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[e + f*x])/(a + b)])/(c*f*Sqrt[a + b*Sin[e + f*x]]) + (Cos[e + f*x]*
Sqrt[a + b*Sin[e + f*x]])/(f*(c + c*Sin[e + f*x]))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3014

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*((c
_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[a/c, Int[1/(Sin[e +
 f*x]*Sqrt[a + b*Sin[e + f*x]]), x], x] + Dist[(b*c - a*d)/c, Int[1/(Sqrt[a
 + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = (-a+b)\int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx + \frac{a\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx}{c}$$

$$= \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c+c\sin(e+fx))} - \frac{b\int \frac{-\frac{c}{2}-\frac{1}{2}c\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx}{c^2} + \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{c\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c+c\sin(e+fx))} + \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{2c} - \frac{(a-b)\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx}{2c}$$

$$= \frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c+c\sin(e+fx))} + \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{2c\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\left((a-b)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{2c\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{cf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{(a-b)\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(c+c\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.05 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.90

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8\sin\left(\frac{1}{2}(e+fx)\right)\sqrt{a+b\sin(e+fx)}+\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-8*Sin[(e + f*x)/2]*Sqrt[a + b*Sin[
e + f*x]] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((-2*I)*(-2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a
 - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e +
 f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)]))*Sec[e + f*x]*Sqrt[-((b
*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[e + f*x]))/(a - b))]/(a
*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Sin[e + f*x]] - (4*b*EllipticF[(-2*e
 + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a
 + b*Sin[e + f*x]] - (2*(4*a + b)*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])))/
(4*c*f*(1 + Sin[e + f*x]))
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.49

method	result
default	$\frac{\sqrt{-(-b\sin(fx+e)-a)(\cos^2(fx+e))}\left(-\frac{2\left(\frac{a}{b}-1\right)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{\frac{(1-\sin(fx+e))b}{a+b}}\sqrt{\frac{(-\sin(fx+e)-1)b}{a-b}}\,b\Pi\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},-\frac{\left(-\frac{a}{b}+1\right)b}{a}\right.}{\sqrt{-(-b\sin(fx+e)-a)\left(\cos^2(fx+e)\right)}}\right.}{}$

```
[In] int((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)/c*(-2*(1/b*a-1)*((a+b*sin(f*x+e))/(
a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1
/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*b*EllipticPi(((a+b*sin(f*x+e))/
(a-b))^(1/2),-(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2))+(-a+b)*(-(-b*sin(f*x+e)^2
-a*sin(f*x+e)+b*sin(f*x+e)+a)/(a-b)/((1+sin(f*x+e))*(sin(f*x+e)-1)*(-b*sin(
```

```
f*x+e)-a))^(1/2)-2*b/(2*a-2*b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/
(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1/2)/(-(-b*sin(f
*x+e)-a)*cos(f*x+e)^2)^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))-b/(a-b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(
1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1/2)/(-(-b*sin(f*x+e)-a
)*cos(f*x+e)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2)))))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \frac{\int \frac{\sqrt{a+b\sin(e+fx)}}{\sin^2(e+fx)+\sin(e+fx)}\,dx}{c}$$

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(sin(e + f*x)**2 + sin(e + f*x)), x)/c
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin(fx+e)+a}}{(c\sin(fx+e)+c)\sin(fx+e)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin(fx+e)+a}}{(c\sin(fx+e)+c)\sin(fx+e)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{a+b\,\sin(e+f\,x)}}{\sin(e+f\,x)\,\left(c+c\,\sin(e+f\,x)\right)}\,dx$$

```
[In] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + c*sin(e + f*x))),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + c*sin(e + f*x))), x)
```

3.30 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 33, antiderivative size = 246

$$\begin{aligned}
&\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx \\
&= \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} \\
&\quad - \frac{\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} \\
&\quad + \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f(c+c\sin(e+fx))}
\end{aligned}$$

```
[Out] cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a-b)/f/(c+c*sin(f*x+e))-(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sin(
f*x+e))/(a+b))^(1/2)+(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1
/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(f*x+e))/(a+b))^(1/2)/c/f/(a+b*sin(f*x+e))^(1/2)-2*(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*
x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/c/f/(a+b*sin(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3020, 2886, 2884, 2847, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$
$$=\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f(a-b)(c\sin(e+fx)+c)}-\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}}$$
$$+\frac{\sqrt{a+b\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$
$$+\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2b}{a+b}\right)}{cf\sqrt{a+b\sin(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x])),x]
```

```
[Out] (EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/((a
 - b)*c*f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - (EllipticF[(e - Pi/2 + f*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(c*f*Sqrt[a + b*Sin[
e + f*x]]) + (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[e + f*x])/(a + b)])/(c*f*Sqrt[a + b*Sin[e + f*x]]) + (Cos[e + f*x]*Sq
rt[a + b*Sin[e + f*x]])/((a - b)*f*(c + c*Sin[e + f*x]))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3020

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c, Int[1/(Sin[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/c, Int[1/(Sqrt[a + b*Sin
[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx}{c} - \int \frac{1}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$

$$= \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f(c+c\sin(e+fx))} - \frac{b\int \frac{-\frac{c}{2}-\frac{1}{2}c\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx}{(a-b)c^2} + \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{c\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f(c+c\sin(e+fx))} - \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx}{2c} + \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{2(a-b)c}$$

$$= \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f(c+c\sin(e+fx))} + \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{2(a-b)c\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{2c\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{cf\sqrt{a+b\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f(c+c\sin(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.88

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8\sin\left(\frac{1}{2}(e+fx)\right)\sqrt{a+b\sin(e+fx)} - \left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

[In] Integrate[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x])),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-8*Sin[(e + f*x)/2]*Sqrt[a + b*Sin[e + f*x]] - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)]))*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Sqrt[a + b*Sin[e + f*x]] + (4*b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]] + (2*(4*a - 3*b)*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])))/(4*(a - b)*c*f*(1 + Sin[e + f*x]))

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 587, normalized size of antiderivative = 2.39

method	result
default	$\frac{\sqrt{-(-b\sin(fx+e)-a)(\cos^2(fx+e))}\left(-\frac{2\left(\frac{a}{b}-1\right)\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\,\sqrt{\frac{(1-\sin(fx+e))b}{a+b}}\,\sqrt{\frac{(-\sin(fx+e)-1)b}{a-b}}\,b\Pi\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},-\frac{\left(-\frac{a}{b}+1\right)b}{a},\sqrt{\right.\right.}{\sqrt{-(-b\sin(fx+e)-a)\left(\cos^2(fx+e)\right)}\,a}\right.}{}$

[In] int(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)/c*(-2*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*b/a*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2))+(-b*sin(f*x+e)^2-a*sin(f*x+e)+b*sin(f*x+e)+a)/(a-b)/((1+sin(f*x+e))*(sin(f*x+e)-1)*(-b*sin(f*x+e)-a))^(1/2)+2*b/(2*a-2*b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+b/(a-b)*(1/b*a-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(1/(a+b)*(1-sin(f*x+e))*b)^(1/2)*(1/(a-b)*(-sin(f*x+e)-1)*b)^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin{(e+fx)}}\sin^2{(e+fx)}+\sqrt{a+b\sin{(e+fx)}}\sin{(e+fx)}}\,dx}{c}$$

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sin(e + f*x)**2 + sqrt(a + b*sin(e + f
*x))*sin(e + f*x)), x)/c
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sin{(fx+e)}+a}(c\sin{(fx+e)}+c)\sin{(fx+e)}}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$
$$=\int \frac{1}{\sqrt{b\sin(fx+e)+a}(c\sin(fx+e)+c)\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$
$$=\int \frac{1}{\sin(e+f\,x)\,\sqrt{a+b\,\sin(e+f\,x)}\,(c+c\,\sin(e+f\,x))}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))), x)
```

3.31 $\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx$

Optimal result 215
Rubi [A] (verified) 215
Mathematica [C] (warning: unable to verify) 217
Maple [C] (warning: unable to verify) 217
Fricas [F] 218
Sympy [F] 218
Maxima [F] 218
Giac [F] 218
Mupad [F(-1)] 219

Optimal result

Integrand size = 39, antiderivative size = 267

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \frac{2\sqrt{g}\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}\sqrt{g\sin(e+fx)}}{\sqrt{g}\sqrt{a+b\sin(e+fx)}}\right),-\frac{a-b}{a+b}\right)\sec(e+fx)\sqrt{\frac{a(1-\sin(e+fx))}{a+b\sin(e+fx)}}\sqrt{\frac{a(1+\sin(e+fx))}{a+b\sin(e+fx)}}(a+b\sin}{\sqrt{a+b}cf} + \frac{gE\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

```
[Out] 2*EllipticPi((a+b)^(1/2)*(g*sin(f*x+e))^(1/2)/g^(1/2)/(a+b*sin(f*x+e))^(1/2
),b/(a+b),((-a+b)/(a+b))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*g^(1/2)*(a*(1-s
in(f*x+e))/(a+b*sin(f*x+e)))^(1/2)*(a*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2
)/c/f/(a+b)^(1/2)+g*EllipticE(cos(f*x+e)/(1+sin(f*x+e)),((-a+b)/(a+b))^(1/2
))*(sin(f*x+e)/(1+sin(f*x+e)))^(1/2)*(a+b*sin(f*x+e))^(1/2)/c/f/(g*sin(f*x+
e))^(1/2)/((a+b*sin(f*x+e))/(a+b)/(1+sin(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used

$= \{3007, 2890, 3011\}$

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx$$
$$= \frac{g\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}}\sqrt{a+b\sin(e+fx)}E\left(\arcsin\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)|-\frac{a-b}{a+b}\right)}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}}$$
$$+\frac{2\sqrt{g}\sec(e+fx)\sqrt{\frac{a(1-\sin(e+fx))}{a+b\sin(e+fx)}}\sqrt{\frac{a(\sin(e+fx)+1)}{a+b\sin(e+fx)}}(a+b\sin(e+fx))\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}\sqrt{g\sin(e+fx)}}{\sqrt{g}\sqrt{a+b\sin(e+fx)}}\right.\right.}{cf\sqrt{a+b}}$$

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[g]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[g*Sin[e + f*x]])/
(Sqrt[g]*Sqrt[a + b*Sin[e + f*x]])], -((a - b)/(a + b))]*Sec[e + f*x]*Sqrt[
(a*(1 - Sin[e + f*x]))/(a + b*Sin[e + f*x])]*Sqrt[(a*(1 + Sin[e + f*x]))/(a
 + b*Sin[e + f*x])]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*c*f) + (g*EllipticE[
ArcSin[Cos[e + f*x]/(1 + Sin[e + f*x])], -((a - b)/(a + b))]*Sqrt[Sin[e + f
*x]/(1 + Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])/(c*f*Sqrt[g*Sin[e + f*x]]
*Sqrt[(a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x]))])
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 3007

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]])/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g/d, In
t[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Sin[e + f*x]], x], x] - Dist[c*(g/d), Int
[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^
2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3011

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
```

```
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
 f*x]))))]))*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
 a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\left(g\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx\right) + \frac{g\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}}\,dx}{c}$$

$$= \frac{2\sqrt{g}\,\text{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{g\sin(e+fx)}}{\sqrt{g}\sqrt{a+b\sin(e+fx)}}\right), -\frac{a-b}{a+b}\right)\sec(e+fx)\sqrt{\frac{a(1-\sin(e+fx))}{a+b\sin(e+fx)}}\sqrt{\frac{a(1+\sin(e+fx))}{a+b\sin(e+fx)}}(a}{\sqrt{a+b}cf}$$

$$+ \frac{gE\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.78 (sec) , antiderivative size = 13199, normalized size of antiderivative = 49.43

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + c*Sin[e + f*
x]),x]
```

```
[Out] Result too large to show
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 9956, normalized size of antiderivative = 37.29

method	result	size
default	Expression too large to display	9956

```
[In] int((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{g\sin\left(fx+e\right)}}{c\sin\left(fx+e\right)+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) + c)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \frac{\int \frac{\sqrt{g\sin\left(e+fx\right)}\sqrt{a+b\sin\left(e+fx\right)}}{\sin\left(e+fx\right)+1}\,dx}{c}$$

```
[In] integrate((g*sin(f*x+e))**(1/2)*(a+b*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))/(sin(e + f*x) + 1),
x)/c
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{g\sin\left(fx+e\right)}}{c\sin\left(fx+e\right)+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) + c
), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{g\sin\left(fx+e\right)}}{c\sin\left(fx+e\right)+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*sin(f*x + e) + c
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+c\sin(e+fx)}\,dx=\int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}{c+c\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x)
),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x)
), x)
```

3.32 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx$

Optimal result . . . 220
Rubi [A] (verified) . . . 220
Mathematica [B] (warning: unable to verify) . . . 221
Maple [B] (warning: unable to verify) . . . 223
Fricas [F] . . . 225
Sympy [F] . . . 225
Maxima [F] . . . 225
Giac [F] . . . 226
Mupad [F(-1)] . . . 226

Optimal result

Integrand size = 39, antiderivative size = 116

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = -\frac{E\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

```
[Out] -EllipticE(cos(f*x+e)/(1+sin(f*x+e)),((-a+b)/(a+b))^(1/2))*(sin(f*x+e)/(1+s
in(f*x+e)))^(1/2)*(a+b*sin(f*x+e))^(1/2)/c/f/(g*sin(f*x+e))^(1/2)/((a+b*sin
(f*x+e))/(a+b)/(1+sin(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {3011}

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = -\frac{\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}}\sqrt{a+b\sin(e+fx)}E\left(\arcsin\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)\middle|-\frac{a-b}{a+b}\right)}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}}$$

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + c*Sin[e + f*x])),x]
```

```
[Out] -((EllipticE[ArcSin[Cos[e + f*x]/(1 + Sin[e + f*x])], -((a - b)/(a + b))]*S
qrt[Sin[e + f*x]/(1 + Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])/(c*f*Sqrt[g*
Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x]))]))
```

Rule 3011

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
 f*x])))]))*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
 a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{E\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3415 vs. $2(116) = 232$.

Time = 28.98 (sec) , antiderivative size = 3415, normalized size of antiderivative = 29.44

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + c*Sin[e + f*x
])),x]
```

```
[Out] (-2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]*Sqr
t[a + b*Sin[e + f*x]])/(f*Sqrt[g*Sin[e + f*x]]*(c + c*Sin[e + f*x])) + ((Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a + b*Sin[e + f*x]]*((a*Sqrt[Sin[
e + f*x]])/(2*Sqrt[a + b*Sin[e + f*x]]) - (b*Sqrt[Sin[e + f*x]])/(2*Sqrt[a
+ b*Sin[e + f*x]]) + (a*Cot[(e + f*x)/2]*Sqrt[Sin[e + f*x]])/(2*Sqrt[a + b*
Sin[e + f*x]]) + (b*Cot[(e + f*x)/2]*Sqrt[Sin[e + f*x]])/(2*Sqrt[a + b*Sin[
e + f*x]]) - (b*Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sqrt[Sin[e + f*x]])/(
2*Sqrt[a + b*Sin[e + f*x]]) + (b*Csc[(e + f*x)/2]*Sqrt[Sin[e + f*x]]*Sin[(3
*(e + f*x))/2])/(2*Sqrt[a + b*Sin[e + f*x]]))*(1 - Cos[e + f*x] + Sin[e + f
*x] - (2*a*(EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/
2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2]
)]*Tan[(e + f*x)/2] + EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(
e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^
```

```
2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan
[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e
+ f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])))/(f*Sqrt[g*Sin[e + f*x]]*(c + c*Sin[e + f*x])*((b*
Cos[e + f*x]*(1 - Cos[e + f*x] + Sin[e + f*x] - (2*a*(EllipticE[ArcSin[Sqrt
[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (
2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2] + EllipticF[A
rcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sq
rt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/
2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b
^2]))]]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))
/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(2*Sqrt
[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Cos[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]]*(1 - Cos[e + f*x] + Sin[e + f*x] - (2*a*(EllipticE[ArcSin[Sqrt[(-b +
 Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt
[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2] + EllipticF[ArcSin[
Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]]
, (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]
]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2
- b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(2*Sin[e + f*
x]^(3/2)) + (Sqrt[a + b*Sin[e + f*x]]*(Cos[e + f*x] + Sin[e + f*x] + (a^2*S
ec[(e + f*x)/2]^2*(EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e
+ f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2
 + b^2])]*Tan[(e + f*x)/2] + EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] +
a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + S
qrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-
((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/(2*Sqrt[-a^2 + b^2]*(-b +
Sqrt[-a^2 + b^2])*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b
^2)]*((a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) + (a*((a*b*Cos[e
 + f*x]*Sec[(e + f*x)/2]^2)/(a^2 - b^2) + (a*Sec[(e + f*x)/2]^2*(a + b*Sin[
e + f*x])*Tan[(e + f*x)/2])/(a^2 - b^2))*(EllipticE[ArcSin[Sqrt[(-b + Sqrt[
-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2
+ b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2] + EllipticF[ArcSin[Sqrt[(
b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*S
qrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sq
rt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/(Sq
rt[-a^2 + b^2]*((a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2))^(3
/2)*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) - (2*a*((EllipticE[
ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/
Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2)
/2 - (a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/S
qrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec
[(e + f*x)/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*(b +
 Sqrt[-a^2 + b^2])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) +
```

```
(a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-
a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e +
 f*x)/2]^2*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*(-b + S
qrt[-a^2 + b^2])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) - (a*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2] - a*Tan[
(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(-
b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (-b +
 Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]) + (a*Sec[(e
+ f*x)/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Ta
n[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[
(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (b +
 Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]*Sqrt[1 - (b +
 Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2])])))/(Sqrt[-a
^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqr
t[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/Sqrt[Sin[e + f*x]]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3006 vs. $2(108) = 216$.

Time = 2.16 (sec) , antiderivative size = 3007, normalized size of antiderivative = 25.92

method	result	size
default	Expression too large to display	3007

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2/c/f/(g/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(csc(f*x+e)-cot(f*x+e))+a)/((1-cos(f
*x+e))^2*csc(f*x+e)^2+1))^(1/2)*((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot
(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot
(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-co
t(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^
(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*a*(csc(f*x+e)-cot(f*x+e))+(1/(b+(-a^
2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^
2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+
(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)
^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*(
(b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a^2*(csc(f*x+e)-cot(f
*x+e))+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+
b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-
b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF
((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1
/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a*b*
```

```
(csc(f*x+e)-cot(f*x+e))+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e
)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^
2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*b*(csc(f*x+e)-cot(f*x+e))-2*(1/(b+(-a^2+b^2
)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2
)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2
+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2
))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-
a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e)
)+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))
^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))
^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1
/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)
,1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*b^2*(cs
c(f*x+e)-cot(f*x+e))+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a
^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a
^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(
1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2
)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)
)*(-a^2+b^2)^(1/2)*2^(1/2)*a+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x
+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*2^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)
-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e
)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f
*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2))*a^2+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-
a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-
a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^
(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^
2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2
))*2^(1/2)*a*b+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b
^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b
^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1
/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-
a^2+b^2)^(1/2)*2^(1/2)*b-2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+
e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a
^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))
^(1/2))*2^(1/2)*a^2+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-
a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-
a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^
(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^
```

```
2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2
))*2^(1/2)*b^2+2*csc(f*x+e)^3*a^2*(1-cos(f*x+e))^3+4*csc(f*x+e)^2*a*b*(1-co
s(f*x+e))^2+2*a^2*(csc(f*x+e)-cot(f*x+e)))/(-cot(f*x+e)+csc(f*x+e)+1)/(a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(csc(f*x+e)-cot(f*x+e))+a)*2^(1/2)/a
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(c\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(c*g*cos(f*x + e)^2
 - c*g*sin(f*x + e) - c*g), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\sqrt{g\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{g\sin\left(e+fx\right)}}\,dx}{c}$$

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(sqrt(g*sin(e + f*x))*sin(e + f*x) + sqrt
(g*sin(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(c\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(c\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((c*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\left(c+c\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((a + b*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))
),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))
), x)
```

3.33 $\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 252

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{gE\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}} - \frac{2\sqrt{a+b}\sqrt{g}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{(a-b)cf}$$

```
[Out] g*EllipticE(cos(f*x+e)/(1+sin(f*x+e)),((-a+b)/(a+b))^(1/2))*(sin(f*x+e)/(1+
sin(f*x+e)))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a-b)/c/f/(g*sin(f*x+e))^(1/2)/((
a+b*sin(f*x+e))/(a+b)/(1+sin(f*x+e)))^(1/2)-2*EllipticF(g^(1/2)*(a+b*sin(f*
x+e))^(1/2)/(a+b)^(1/2)/(g*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*g^(1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*t
an(f*x+e)/(a-b)/c/f
```

Rubi [A] (verified)

Time = 0.35 (sec), antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used

$= \{3015, 2895, 3011\}$

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$

$$= \frac{g\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}}\sqrt{a+b\sin(e+fx)}E\left(\arcsin\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)|-\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}} - \frac{2\sqrt{g}\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)}{cf(a-b)}$$

```
[In] Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x])),x]
```

```
[Out] (g*EllipticE[ArcSin[Cos[e + f*x]/(1 + Sin[e + f*x])], -((a - b)/(a + b))]*S
qrt[Sin[e + f*x]/(1 + Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])/((a - b)*c*f
*Sqrt[g*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x])
)]) - (2*Sqrt[a + b]*Sqrt[g]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(
1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[g]*Sqrt[a + b*Sin[e + f*
x]])/(Sqrt[a + b]*Sqrt[g*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])
/((a - b)*c*f)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3011

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
 f*x])))]))*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
 a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3015

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-a)*(g
/(b*c - a*d)), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x
] + Dist[c*(g/(b*c - a*d)), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*
```

```
x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{g\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx}{a-b} + \frac{(ag)\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx}{(a-b)c}$$

$$= \frac{gE\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}}$$

$$-\frac{2\sqrt{a+b}\sqrt{g}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+}{(a-b)cf}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4464 vs. $2(252) = 504$.

Time = 28.85 (sec) , antiderivative size = 4464, normalized size of antiderivative = 17.71

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x
])),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[g*Sin[e + f*
x]]*Sqrt[a + b*Sin[e + f*x]])/((a - b)*f*(c + c*Sin[e + f*x])) + (Cot[(e +
f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[g*Sin[e + f*x]]*Sqrt[a
 + b*Sin[e + f*x]]*(-1/2*(a*Sqrt[Sin[e + f*x]])/((a - b)*Sqrt[a + b*Sin[e +
 f*x]]) - (b*Sqrt[Sin[e + f*x]])/(2*(a - b)*Sqrt[a + b*Sin[e + f*x]]) - (b*
Cos[(3*(e + f*x))/2]*Sec[(e + f*x)/2]*Sqrt[Sin[e + f*x]])/(2*(a - b)*Sqrt[a
 + b*Sin[e + f*x]]) - (b*Sec[(e + f*x)/2]*Sqrt[Sin[e + f*x]]*Sin[(3*(e + f*
x))/2])/(2*(a - b)*Sqrt[a + b*Sin[e + f*x]]) + (a*Sqrt[Sin[e + f*x]]*Tan[(e
 + f*x)/2])/(2*(a - b)*Sqrt[a + b*Sin[e + f*x]]) - (b*Sqrt[Sin[e + f*x]]*Ta
n[(e + f*x)/2])/(2*(a - b)*Sqrt[a + b*Sin[e + f*x]]))*(-2*Tan[(e + f*x)/2]*
(1 + Tan[(e + f*x)/2]) + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(a^2 - b^2)]*(-(EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 +
b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])
/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + EllipticF[ArcSin[Sqrt[(b + Sq
rt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a
^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^
2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*S
```

```
in[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]])))/(2*(a -
b)*f*(c + c*Sin[e + f*x])*((b*Cos[e + f*x]*Cot[(e + f*x)/2]*Sqrt[Sin[e + f*
x]]*(-2*Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]) + (2*Sqrt[-a^2 + b^2]*Sqrt[
(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(EllipticE[ArcSi
n[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[
2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + Elli
pticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b
^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e
+ f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-
a^2 + b^2]))]]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[
-a^2 + b^2])])))/(4*(a - b)*Sqrt[a + b*Sin[e + f*x]]) + (Cos[e + f*x]*Cot[(
e + f*x)/2]*Sqrt[a + b*Sin[e + f*x]]*(-2*Tan[(e + f*x)/2]*(1 + Tan[(e + f*x
)/2]) + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x])
)/(a^2 - b^2)]*(-(EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e +
 f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2
+ b^2])]*Tan[(e + f*x)/2]) + EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] +
a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + S
qrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-
((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/((a + b*Sin[e + f*x])*Sqrt
[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(4*(a - b)*Sqrt[Sin[e + f
*x]]) - (Csc[(e + f*x)/2]^2*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(-2
*Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]) + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[
(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(EllipticE[ArcSin[Sqrt[
(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2
*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + EllipticF[A
rcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sq
rt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/
2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b
^2]))]]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 +
b^2])])))/(4*(a - b)) + (Cot[(e + f*x)/2]*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin
[e + f*x]]*(-(Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - Sec[(e + f*x)/2]^2*(1
+ Tan[(e + f*x)/2]) - (a*Sqrt[-a^2 + b^2]*Sec[(e + f*x)/2]^2*Sqrt[(a*Sec[(e
 + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(EllipticE[ArcSin[Sqrt[(-
b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*S
qrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2]
)/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2
]))]]))/(2*(-b + Sqrt[-a^2 + b^2])*(a + b*Sin[e + f*x])*((a*Tan[(e + f*x)/2]
)/(-b + Sqrt[-a^2 + b^2]))^(3/2)) - (2*b*Sqrt[-a^2 + b^2]*Cos[e + f*x]*Sqrt
[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(EllipticE[ArcS
in[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + Ell
ipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e
```

```
 + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[
-a^2 + b^2]))]))/((a + b*Sin[e + f*x])^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sq
rt[-a^2 + b^2])]) + (Sqrt[-a^2 + b^2]*((a*b*Cos[e + f*x]*Sec[(e + f*x)/2]^2
)/(a^2 - b^2) + (a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]
)/(a^2 - b^2))*(-(EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e +
 f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2
+ b^2])]*Tan[(e + f*x)/2]) + EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] +
a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + S
qrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-
((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt
[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e +
f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f
*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-1/2*(EllipticE[ArcSin[Sqrt[(-
b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*S
qrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2) - (a*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]
/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2*
Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*(b + Sqrt[-a^2 + b^2
])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2*Sqrt[
-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*(-b + Sqrt[-a^2 + b^2])
*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (a*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(-b + Sqrt[-a^2 +
b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2
] - a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]) + (a*Sec[(e + f*x)/2]^2*Sqrt
[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/
(b + Sqrt[-a^2 + b^2]))])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(b + Sqrt[-a^2 +
 b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (b + Sqrt[-a^2 + b^2
] + a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]*Sqrt[1 - (b + Sqrt[-a^2 + b^2
] + a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2])])))/((a + b*Sin[e + f*x])*Sq
rt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(2*(a - b))))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3056 vs. 2(232) = 464.

Time = 2.48 (sec) , antiderivative size = 3057, normalized size of antiderivative = 12.13

method	result	size
default	Expression too large to display	3057

```
[In] int((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2/c/f*(g/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(c
sc(f*x+e)-cot(f*x+e))+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))^(1/2)*((1/(b+(-
a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-
a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(
b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^
2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)
*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*a*
(csc(f*x+e)-cot(f*x+e))-2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e
)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^
2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*b*(csc(f*x+e)-cot(f*x+e))-(1/(b+(-a^2+b^2)^
(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^
(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b
^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))
*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^
2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+
(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/
2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/
2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+
(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2
*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a*b*(csc(f*
x+e)-cot(f*x+e))+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2
+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2
+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/
2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^
(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*
2^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))-2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e
)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e
)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+
e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(
f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+
b^2)^(1/2))^(1/2))*2^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e))+(1/(b+(-a^2+b^2)^(1/
2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/
2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)
^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a
*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b
^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*a-2*(1/(b+(-a^
2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^
2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+
(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)
^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*(
(b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*b-(1
```

```
/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)
*2^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b
))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF(
(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/
2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2+(1/(b+(-a
^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a
^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b
+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2
)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*
((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a*b+2*(1/(b+(-a^2+b^
2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^
2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^
2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/
2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(
-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a^2-2*(1/(b+(-a^2+b^2)^(1
/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1
/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2
)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(
a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+
b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*b^2-2*csc(f*x+e)^3*a^2*(1-cos(
f*x+e))^3-4*csc(f*x+e)^2*a*b*(1-cos(f*x+e))^2-2*a^2*(csc(f*x+e)-cot(f*x+e))
)/(-cot(f*x+e)+csc(f*x+e)+1)/(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(csc(f*x+
e)-cot(f*x+e))+a)/(1-cos(f*x+e))*sin(f*x+e)*2^(1/2)/(a-b)/a
```

Fricas [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx=\int \frac{\sqrt{g\sin\left(fx+e\right)}}{\sqrt{b\sin\left(fx+e\right)+a}(c\sin\left(fx+e\right)+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(b*c*cos(f*x + e)^2
 - (a + b)*c*sin(f*x + e) - (a + b)*c), x)
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\sin{(e+fx)}}}{\sqrt{a+b\sin{(e+fx)}}\sin{(e+fx)}+\sqrt{a+b\sin{(e+fx)}}}\,dx}{c}$$

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))/(sqrt(a + b*sin(e + f*x))*sin(e + f*x) + sqrt
(a + b*sin(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(fx+e)}}}{\sqrt{b\sin{(fx+e)}+a}(c\sin{(fx+e)}+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(fx+e)}}}{\sqrt{b\sin{(fx+e)}+a}(c\sin{(fx+e)}+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,\left(c+c\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))
),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))
), x)
```

3.34 $\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$

Optimal result 236
Rubi [A] (verified) 236
Mathematica [C] (warning: unable to verify) 238
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Fricas [F] 242
Sympy [F] 242
Maxima [F] 243
Giac [F] 243
Mupad [F(-1)] 243

Optimal result

Integrand size = 39, antiderivative size = 256

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = -\frac{E\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}} + \frac{2b\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{a(a-b)cf\sqrt{g}}$$

```
[Out] -EllipticE(cos(f*x+e)/(1+sin(f*x+e)),((-a+b)/(a+b))^(1/2))*(sin(f*x+e)/(1+s
in(f*x+e)))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a-b)/c/f/(g*sin(f*x+e))^(1/2)/((a
+b*sin(f*x+e))/(a+b)/(1+sin(f*x+e)))^(1/2)+2*b*EllipticF(g^(1/2)*(a+b*sin(f
*x+e))^(1/2)/(a+b)^(1/2)/(g*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(
1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+
e)/a/(a-b)/c/f/g^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{3017, 2895, 3011\}$

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$

$$= \frac{2b\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)}{acf\sqrt{g}(a-b)} - \frac{\sqrt{\frac{\sin(e+fx)}{\sin(e+fx)+1}}\sqrt{a+b\sin(e+fx)}E\left(\arcsin\left(\frac{\cos(e+fx)}{\sin(e+fx)+1}\right)|-\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(\sin(e+fx)+1)}}}$$

```
[In] Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f*x])),
x]
```

```
[Out] -((EllipticE[ArcSin[Cos[e + f*x]/(1 + Sin[e + f*x])], -((a - b)/(a + b))]*S
qrt[Sin[e + f*x]/(1 + Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])/((a - b)*c*f
*Sqrt[g*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x])
)])) + (2*b*Sqrt[a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + C
sc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[g]*Sqrt[a + b*Sin[e + f*x]])/
(Sqrt[a + b]*Sqrt[g*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(a*(
a - b)*c*f*Sqrt[g])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3011

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(-Sqrt[
a + b*Sin[e + f*x]])*(Sqrt[d*(Sin[e + f*x]/(c + d*Sin[e + f*x]))]/(d*f*Sqrt
[g*Sin[e + f*x]]*Sqrt[c^2*((a + b*Sin[e + f*x])/((a*c + b*d)*(c + d*Sin[e +
 f*x])))]))*EllipticE[ArcSin[c*(Cos[e + f*x]/(c + d*Sin[e + f*x]))], (b*c -
 a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3017

```
Int[1/(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*
c - a*d), Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - D
```

```
ist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c +
d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*
d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{b\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx}{(a-b)c} - \frac{c\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}\,dx}{-ac+bc}$$
$$= -\frac{E\left(\arcsin\left(\frac{\cos(e+fx)}{1+\sin(e+fx)}\right)\middle|-\frac{a-b}{a+b}\right)\sqrt{\frac{\sin(e+fx)}{1+\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}{(a-b)cf\sqrt{g\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a+b)(1+\sin(e+fx))}}} + \frac{2b\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{a(a-b)cf\sqrt{g}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.75 (sec) , antiderivative size = 1667, normalized size of antiderivative = 6.51

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx$$
$$= -\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)\sqrt{a+b\sin(e+fx)}}{(a-b)f\sqrt{g\sin(e+fx)}(c+c\sin(e+fx))}$$
$$+ \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{\sin(e+fx)}\left(\frac{4a(a-b)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)}{-a+b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\csc^2\left(\frac{1}{2}\left(-e \right.\right.}}{}\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + c*Sin[e + f
*x])),x]
```

```
[Out] (-2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]*Sqr
t[a + b*Sin[e + f*x]])/((a - b)*f*Sqrt[g*Sin[e + f*x]]*(c + c*Sin[e + f*x])
```

```
) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[Sin[e + f*x]]*((4*a*(a -
b)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) + (2
*a*ArcTanh[(Sqrt[b]*Sqrt[Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]]*Cos[e + f
*x]^2)/(Sqrt[b]*(1 - Sin[e + f*x]^2)) + 4*a^2*((Sqrt[((a + b)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a
)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt
[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*
x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[
Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) - 2*b*((Cos[e + f*x]*Sqrt[a + b*Si
n[e + f*x]])/(b*Sqrt[Sin[e + f*x]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f
*x]*EllipticE[I*ArcSinh[Sin[(-e + Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a
)/(-a - b)]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Cs
c[e + f*x]]*Sqrt[(Csc[e + f*x]*(a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*S
qrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqr
t[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[
Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b
*Sin[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])))/b) +
(2*b*Cot[e + f*x]*(-((a*ArcTanh[(Sqrt[b]*Sqrt[Sin[e + f*x]])/(-Sqrt[a] + Sq
rt[a + b*Sin[e + f*x]])])/b^(3/2)) + (Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(2*b))*Sin[2*(e + f*x)])/(1 - Sin[e + f*x]^2)))/(2*(a - b)*f*Sqrt[g
*Sin[e + f*x]]*(c + c*Sin[e + f*x]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3937 vs. 2(236) = 472.

Time = 2.61 (sec) , antiderivative size = 3938, normalized size of antiderivative = 15.38

method	result	size
default	Expression too large to display	3938

```
[In] int(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/c/f/(g/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(csc(f*x+e)-cot(f*x+e))+a)/((1-cos(f
*x+e))^2*csc(f*x+e)^2+1))^(1/2)*((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot
(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot
(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-co
t(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^
(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*a*(csc(f*x+e)-cot(f*x+e))-2*2^(1/2)*
(-a^2+b^2)^(1/2)*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b
^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b
^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1
/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b*
(csc(f*x+e)-cot(f*x+e))+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e)
)+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e
)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^
2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*(-a^2+b^2)^(1/2)*2^(1/2)*b*(csc(f*x+e)-cot(f*x+e))+(1/(b+(-a^2+b^2)^
(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^
(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b
^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))
*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^
2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+
(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/
2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/
2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+
(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2
*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a*b*(csc(f*
x+e)-cot(f*x+e))-2*2^(1/2)*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+
e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a
^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))
```

```
^(1/2))*b^2*(csc(f*x+e)-cot(f*x+e))-2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e
)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e
)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+
e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(
f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+
b^2)^(1/2))^(1/2))*2^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+2*(1/(b+(-a^2+b^2)^(
1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(
1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^
2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*
(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2
+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e))+(
1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2
)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2
)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1/(b+(
-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*2^(
1/2)*a-2*2^(1/2)*(-a^2+b^2)^(1/2)*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-co
t(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-co
t(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-c
ot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+
e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)
^(1/2))^(1/2))*b+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2
+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2
+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/
2)*EllipticE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^
(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*
(-a^2+b^2)^(1/2)*2^(1/2)*b+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e
))+(-a^2+b^2)^(1/2)+b))^(1/2)*2^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-c
ot(f*x+e))+(-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-
cot(f*x+e)))^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x
+e))+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2
)^(1/2))^(1/2))*a^2+(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^
2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^
2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1
/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)
^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))
*2^(1/2)*a*b-2*2^(1/2)*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(
-a^2+b^2)^(1/2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(
-a^2+b^2)^(1/2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))
^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b
^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/
2))*b^2-2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/
2)+b))^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/
2)-b))^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellipt
icE((1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))
```

```
^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*a
^2+2*(1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b)
)^(1/2)*(1/(-a^2+b^2)^(1/2)*(-a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)-b)
)^(1/2)*(-a/(b+(-a^2+b^2)^(1/2))*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((
1/(b+(-a^2+b^2)^(1/2))*(a*(csc(f*x+e)-cot(f*x+e))+(-a^2+b^2)^(1/2)+b))^(1/2
),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*2^(1/2)*b^2+2*
csc(f*x+e)^3*a^2*(1-cos(f*x+e))^3+4*csc(f*x+e)^2*a*b*(1-cos(f*x+e))^2+2*a^2
*(csc(f*x+e)-cot(f*x+e)))/(-cot(f*x+e)+csc(f*x+e)+1)/(a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+2*b*(csc(f*x+e)-cot(f*x+e))+a)*2^(1/2)/a/(a-b)
```

Fricas [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sin{(fx+e)}+a}(c\sin{(fx+e)}+c)\sqrt{g\sin{(fx+e)}}}\,dx$$

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/((a + b)*c*g*cos(f*
x + e)^2 - (a + b)*c*g + (b*c*g*cos(f*x + e)^2 - (a + b)*c*g)*sin(f*x + e))
, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\sin{(e+fx)}}\sqrt{a+b\sin{(e+fx)}}\sin{(e+fx)}+\sqrt{g\sin{(e+fx)}}\sqrt{a+b\sin{(e+fx)}}}\,dx}{c}$$

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))*sin(e + f*x) + sq
rt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sin(fx+e)+a}(c\sin(fx+e)+c)\sqrt{g\sin(fx+e)}}\,dx$$

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sqrt(g*sin(f*x +
 e))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sin(fx+e)+a}(c\sin(fx+e)+c)\sqrt{g\sin(fx+e)}}\,dx$$

```
[In] integrate(1/(c+c*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
 algorithm="giac")
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(c*sin(f*x + e) + c)*sqrt(g*sin(f*x +
 e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+c\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{g\,\sin(e+f\,x)}\,\sqrt{a+b\,\sin(e+f\,x)}\,(c+c\,\sin(e+f\,x))}\,dx$$

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x
))),x)
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x
))), x)
```

3.35 $\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal result 244
Rubi [A] (verified) 244
Mathematica [C] (warning: unable to verify) 246
Maple [B] (warning: unable to verify) 247
Fricas [B] (verification not implemented) 247
Sympy [F] 249
Maxima [F] 249
Giac [F(-1)] 250
Mupad [F(-1)] 250

Optimal result

Integrand size = 35, antiderivative size = 123

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{2\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*c^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e
))^(1/2))*a^(1/2)*c^(1/2)/f-2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*a^(1/2)*d^(1/2)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3028, 2854, 211, 3022, 212}

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{2\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

```
[In] Int[Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/f - (2*Sqrt[a]*Sqrt[c]*ArcTanh[(Sqrt[a
]*Sqrt[c]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
])/f
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 3022

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst
[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

Rule 3028

```
Int[(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]])/sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[d, Int[Sqrt[a +
 b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[c, Int[Sqrt[a + b*
Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] || NeQ[c^2 -
d^2, 0])
```

Rubi steps

$$\text{integral} = c\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx + d\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= -\frac{(2ac)\text{Subst}\left(\int \frac{1}{1-acx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{(2ad)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{2\sqrt{a}\sqrt{c}\text{carctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.61

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx =$$

$$-\frac{\left(\sqrt{c}\log\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-\frac{ie}{2}}\left(-\sqrt{2}c\left(-1+e^{i(e+fx)}\right)-i\sqrt{2}d\left(1+e^{i(e+fx)}\right)+2i\sqrt{c}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}\right)f}{c^{3/2}\left(1+e^{i(e+fx)}\right)}\right)+\sqrt{c}\log\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-}\ldots}{\ldots}\right.\right.}{\ldots}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(((Sqrt[c]*Log[((1/2 + I/2)*(-(Sqrt[2]*c*(-1 + E^(I*(e + f*x)))) - I*Sqrt[
2]*d*(1 + E^(I*(e + f*x))) + (2*I)*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(
-1 + E^((2*I)*(e + f*x)))])*f)/(c^(3/2)*E^((I/2)*e)*(1 + E^(I*(e + f*x))))]
 + Sqrt[c]*Log[((1/2 + I/2)*((-I)*Sqrt[2]*d*(-1 + E^(I*(e + f*x))) + Sqrt[2
]*c*(1 + E^(I*(e + f*x))) + 2*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 +
E^((2*I)*(e + f*x)))])*f)/(c^(3/2)*E^((I/2)*e)*(-1 + E^(I*(e + f*x))))] - I
*Sqrt[d]*(Log[(2*((-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*S
qrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/(d^(3/2)*E^((
I/2)*(e + 2*f*x)))] - Log[((1 + I)*Sqrt[2]*(c - I*d*Cos[e + f*x] + d*Sin[e
+ f*x] + (1 - I)*Sqrt[d]*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e
+ f*x])]))/Sqrt[d]]))*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*Sqrt[a*(1 + S
in[e + f*x])]*Sqrt[c + d*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5065 vs. $2(99) = 198$.

Time = 0.84 (sec) , antiderivative size = 5066, normalized size of antiderivative = 41.19

output too large to display

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(99) = 198$.

Time = 1.02 (sec) , antiderivative size = 3539, normalized size of antiderivative = 28.77

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^
4)*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (
31*a*c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4
- 2*(81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x +
 e)^3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f
*x + e)^2 - 8*((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 -
14*c^2*d + 5*c*d^2)*cos(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2
*(18*c^3 - 33*c^2*d + 12*c*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d
 + 5*c*d^2)*cos(f*x + e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3
- 51*c^3 + 59*c^2*d - 17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)
*cos(f*x + e)^2 - (25*c^3 - 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a
*c^4 - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*
c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 7
0*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d +
7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^
2*d^2 - 12*a*c*d^3 + a*d^4)*cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a
*c^2*d^2 - a*c*d^3)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x +
 e)^4 - 2*cos(f*x + e)^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x +
 e)^2 + 1)*sin(f*x + e) + cos(f*x + e) + 1)) + sqrt(-a*d)*log((128*a*d^4*co
s(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2
*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)
*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x
+ e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 +
```

17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e
)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x +
e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e
)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*d
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4 - 28*a*c^3*d +
230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x
+ e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^
3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x
+ e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*si
n(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, 1/4*(2*sqrt(-a*c)*arctan
(-1/4*((c^2 - 6*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 -
c*d)*sin(f*x + e))*sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/((a*c^2*d - a*c*d^2)*cos(f*x + e)^3 - (a*c^3 - 3*a*c^2*d)*cos(f*x + e)
*sin(f*x + e) + (2*a*c^3 - a*c^2*d + a*c*d^2)*cos(f*x + e))) + sqrt(-a*d)*l
og((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
+ a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c
*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4
*a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*
x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*
d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (1
6*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*
d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^
4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (1
28*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d
^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5
*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*
cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, 1/4*(2*s
qrt(a*d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
d^2)*sin(f*x + e))*sqrt(a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c)/(2*a*d^3*cos(f*x + e)^3 - (3*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e)
- (a*c^2*d - a*c*d^2 + 2*a*d^3)*cos(f*x + e))) + sqrt(a*c)*log(((a*c^4 - 2
8*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a*c^4 + 4*a
*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a*c^4 - 196*a*c^3*d + 154*a*
c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2*(81*a*c^4 - 252*a*c^3*d + 1
50*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^3 + 2*(79*a*c^4 - 100*a*c^3
*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^2*d +
7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e
)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c*d^2
- d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) + ((c^
3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c*d^2
+ d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3 - 31*
c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a*c^4 - 476*a*c^3*d + 230*a*c^2*
d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +

```
 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^
4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x
+ e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a*d^4)*cos
(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e
))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)^3 - 2*co
s(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e) + cos(f
*x + e) + 1)))/f, 1/2*(sqrt(-a*c)*arctan(-1/4*((c^2 - 6*c*d + d^2)*cos(f*x
+ e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*sqrt(-a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((a*c^2*d - a*c*d^2)*cos(f*x +
 e)^3 - (a*c^3 - 3*a*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*a*c^3 - a*c^2*d
+ a*c*d^2)*cos(f*x + e))) + sqrt(a*d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^
2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*d)*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*cos(f*x + e)^3 - (3*a*c*d^2 - a*d
^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d - a*c*d^2 + 2*a*d^3)*cos(f*x + e))
))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{c+d\sin\left(e+fx\right)}}{\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2)/sin(f*x+e),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))/sin(e + f*x),
x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x
)
```

Giac [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x),x)
```

```
[Out] int(((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x), x
)
```

3.36 $\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result . . . 251
Rubi [A] (verified) . . . 251
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Maple [B] (verified) . . . 253
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Sympy [F] . . . 254
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Giac [F] . . . 254
Mupad [F(-1)] . . . 255

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*c^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e
))^(1/2))*a^(1/2)/f/c^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3022, 212}

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{c}f}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[c]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[c]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3022

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst
[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2a)\text{Subst}\left(\int \frac{1}{1-acx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{c}f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 367, normalized size of antiderivative = 6.02

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx =$$

$$-\frac{\left(\log\left(-\frac{(1+i)e^{\frac{ie}{2}}\left(\sqrt{2}c(-1+e^{i(e+fx)})+i\sqrt{2}d(1+e^{i(e+fx)})-2i\sqrt{c}\sqrt{2ce^{i(e+fx)}-id(-1+e^{2i(e+fx)})}\right)f}{\sqrt{c}(1+e^{i(e+fx)})}\right)+\log\left(\frac{(1+i)e^{\frac{ie}{2}}\left(-i\sqrt{2}d(-\cdots\right.}{\cdots}\right.\cdots}{\cdots}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]],
x]
```

```
[Out] -(((Log[((-1 - I)*E^((I/2)*e)*(Sqrt[2]*c*(-1 + E^(I*(e + f*x))) + I*Sqrt[2]
*d*(1 + E^(I*(e + f*x))) - (2*I)*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
 + E^((2*I)*(e + f*x)))])*f)/(Sqrt[c]*(1 + E^(I*(e + f*x))))] + Log[((1 + I
)*E^((I/2)*e)*((-I)*Sqrt[2]*d*(-1 + E^(I*(e + f*x))) + Sqrt[2]*c*(1 + E^(I*
(e + f*x))) + 2*Sqrt[c]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e +
f*x)))])*f)/(Sqrt[c]*(-1 + E^(I*(e + f*x))))])*(Cos[(e + f*x)/2] - I*Sin[(e
 + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])
*(c + d*Sin[e + f*x])])/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sq
rt[c + d*Sin[e + f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(49) = 98$.

Time = 2.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.33

method	result
default	$\frac{\sqrt{a(1+\sin(fx+e))}\,\sqrt{c+d\sin(fx+e)}\left(\ln\left(-\frac{-\sqrt{c}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}+c\cot(fx+e)-c\csc(fx+e)-d}{\sqrt{c}}\right)-\ln\left(-\frac{2\left(\sqrt{c}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}\ \sin\right.}{\mathrm{co}}\right.\right.}{f(\cos(fx+e)+\sin(fx+e)+1)\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}\,\sqrt{c}}$

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(ln(-(-c^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)+c*cot(f*x+e)-c*csc(f*x+e)-d)/c^(1/2
))-ln(-2*(c^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e
)+c*sin(f*x+e)-cos(f*x+e)*d+d)/(cos(f*x+e)-1)))*2^(1/2)/(cos(f*x+e)+sin(f*x
+e)+1)/((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)/c^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(49) = 98$.

Time = 0.46 (sec) , antiderivative size = 1044, normalized size of antiderivative = 17.11

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*sqrt(a/c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4
)*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (3
1*a*c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 -
 2*(81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x +
e)^3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*
x + e)^2 - 8*((c^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^4 + 51*c^4 -
 59*c^3*d + 17*c^2*d^2 - c*d^3 - 2*(5*c^4 - 14*c^3*d + 5*c^2*d^2)*cos(f*x +
 e)^3 - 2*(18*c^4 - 33*c^3*d + 12*c^2*d^2 - c*d^3)*cos(f*x + e)^2 + 2*(13*c
^4 - 14*c^3*d + 5*c^2*d^2)*cos(f*x + e) - (51*c^4 - 59*c^3*d + 17*c^2*d^2 -
 c*d^3 - (c^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^3 - (11*c^4 - 35*
c^3*d + 17*c^2*d^2 - c*d^3)*cos(f*x + e)^2 + (25*c^4 - 31*c^3*d + 7*c^2*d^2
 - c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c)*sqrt(a/c) + (289*a*c^4 - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*
d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
```

```
a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x +
e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e)^3 - 2*(6
5*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a*d^4)*cos(f*x + e)^2 -
 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*cos(f*x + e))*sin(f*x +
e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)^3 - 2*cos(f*x + e)^2
+ (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e) + cos(f*x + e) + 1))
/f, 1/2*sqrt(-a/c)*arctan(-1/4*((c^2 - 6*c*d + d^2)*cos(f*x + e)^2 - 9*c^2
+ 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d
*sin(f*x + e) + c)*sqrt(-a/c)/((a*c*d - a*d^2)*cos(f*x + e)^3 - (a*c^2 - 3*
a*c*d)*cos(f*x + e)*sin(f*x + e) + (2*a*c^2 - a*c*d + a*d^2)*cos(f*x + e)))
/f]
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(sqrt(c + d*sin(e + f*x))*sin(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(sqrt(d*sin(f*x + e) + c)*sin(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))^(1/2)),x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))^(1/2)), x
)
```

3.37 $\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result 256
Rubi [A] (verified) 256
Mathematica [B] (warning: unable to verify) 258
Maple [B] (warning: unable to verify) 259
Fricas [B] (verification not implemented) 259
Sympy [F] 261
Maxima [F] 261
Giac [F(-1)] 261
Mupad [F(-1)] 262

Optimal result

Integrand size = 35, antiderivative size = 140

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\sqrt{2}\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*c^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e
))^(1/2))*c^(1/2)/f/a^(1/2)+arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^
(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3023, 2861, 214, 3022, 212}

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[(Sqrt[a]*Sqrt[c]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) + (Sqrt[2]*Sqrt[c - d]*ArcTanh[(S
qrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])])/(Sqrt[a]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3022

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst
[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

Rule 3023

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(b*c - a*d)/c,
Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[a/
c, Int[Sqrt[c + d*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$= \frac{c\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a} + (-c+d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= -\frac{(2c)\text{Subst}\left(\int \frac{1}{1-acx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} + \frac{(2a(c-d))\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\sqrt{2}\sqrt{c-d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 665 vs. $2(140) = 280$.

Time = 10.59 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.75

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\csc(e+fx)\left(\sqrt{c}\log\left(\tan\left(\frac{1}{2}(e+fx)\right)\right) - \sqrt{2}\sqrt{c-d}\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right) + \sqrt{c}\log\left(d+\sqrt{2}\sqrt{c}\sqrt{\frac{1}{1+\cos\ldots}}\right.\right.}{f\sqrt{a(1+\sin(e+fx))}\left(\sqrt{c}\csc(e+\ldots\right.}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] (Csc[e + f*x]*(Sqrt[c]*Log[Tan[(e + f*x)/2]] - Sqrt[2]*Sqrt[c - d]*Log[1 +
Tan[(e + f*x)/2]] + Sqrt[c]*Log[d + Sqrt[2]*Sqrt[c]*Sqrt[(1 + Cos[e + f*x])
^(-1)]*Sqrt[c + d*Sin[e + f*x]] + c*Tan[(e + f*x)/2]] - Sqrt[c]*Log[c + Sqr
t[2]*Sqrt[c]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + d*Tan
[(e + f*x)/2]] + Sqrt[2]*Sqrt[c - d]*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]])*Sq
rt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]*(Sqrt[c]*Csc[e + f*x]
 + (c*Sqrt[Sec[(e + f*x)/2]^2])/(2*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[c - d]
*Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[c]*(d*Sec[(e
+ f*x)/2]^2 + (Sqrt[2]*Sqrt[c]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e
 + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]]))/(2*(c + Sqrt[2]*Sqrt[
c]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + d*Tan[(e + f*x)
/2])) + (Sqrt[2]*Sqrt[c - d]*(-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c -
 d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/
Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(
-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(113) = 226$.

Time = 1.80 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{c+d\sin(fx+e)}\,\sqrt{2}\left(\sqrt{2c-2d}\,\ln\left(\frac{2\sqrt{2c-2d}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}\,\sin(fx+e)+2c\sin(fx+e)-2d\sin(fx+e)+2c\cos(fx+e)-2\cos(fx+e)d-2c+2}{-\cos(fx+e)+1+\sin(fx+e)}\right.\right.}{2f(1+\cos(}$

[In] int((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/f*(c+d*sin(f*x+e))^(1/2)*2^(1/2)*((2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-cos(f*x+e)*d-c+d)/(-cos(f*x+e)+1+sin(f*x+e)))*c^(1/2)+ln((c^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)+c*csc(f*x+e)-c*cot(f*x+e)+d)/c^(1/2))*c-c*ln(-2*(c^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-cos(f*x+e)*d+d)/(cos(f*x+e)-1)))*(cos(f*x+e)+sin(f*x+e)+1)/(1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)/c^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(113) = 226$.

Time = 0.61 (sec) , antiderivative size = 2791, normalized size of antiderivative = 19.94

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Too large to display}$$

[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*sqrt((c - d)/a)*log(((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 + 4*sqrt(2)*((c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a) - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) + sqrt(c/a)*log(((c^4 - 28*c^3*d + 70*c^2*d^2 - 28*c*d^3 + d^4)*cos(f*x + e)^5 - (31*c^4 - 196*c^3*d + 154*c^2*d^2 - 4*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(81*c^4 - 252*c^3*d

```
 + 150*c^2*d^2 - 28*c*d^3 + d^4)*cos(f*x + e)^3 + 2*(79*c^4 - 100*c^3*d + 7
4*c^2*d^2 - 4*c*d^3 - d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^2*d + 7*c*d^2 - d
^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e)^3 + 51*c^3
 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c*d^2 - d^3)*cos(f
*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) + ((c^3 - 7*c^2*d
+ 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c*d^2 + d^3 + (11*
c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3 - 31*c^2*d + 7*c*
d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(c/a) + (289*c^4 - 476*c^3*d + 230*c^2*d^2 - 28*c*d^3 + d
^4)*cos(f*x + e) + ((c^4 - 28*c^3*d + 70*c^2*d^2 - 28*c*d^3 + d^4)*cos(f*x
+ e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 + 32*(c^4 - 7*c^3*d + 7*
c^2*d^2 - c*d^3)*cos(f*x + e)^3 - 2*(65*c^4 - 140*c^3*d + 38*c^2*d^2 - 12*c
*d^3 + d^4)*cos(f*x + e)^2 - 32*(9*c^4 - 15*c^3*d + 7*c^2*d^2 - c*d^3)*cos(
f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)^3
 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e)
+ cos(f*x + e) + 1)))/f, 1/4*(sqrt(2)*sqrt((c - d)/a)*log(((c^2 - 14*c*d +
17*d^2)*cos(f*x + e)^3 + 4*sqrt(2)*((c - 3*d)*cos(f*x + e)^2 - (3*c - d)*co
s(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*c + 4*d)
*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a) - (13*c^
2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*
c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2
 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(
cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*s
in(f*x + e) - 2*cos(f*x + e) - 4)) + 2*sqrt(-c/a)*arctan(-1/4*((c^2 - 6*c*d
 + d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-c/a)/((c^2*d - c*d^
2)*cos(f*x + e)^3 - (c^3 - 3*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*c^3 - c^
2*d + c*d^2)*cos(f*x + e))))/f, 1/4*(2*sqrt(2)*sqrt(-(c - d)/a)*arctan(1/4*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*
sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c*d - d^2)*cos(f*x + e)*sin(f*x + e) +
 (c^2 - c*d)*cos(f*x + e))) + sqrt(c/a)*log(((c^4 - 28*c^3*d + 70*c^2*d^2 -
 28*c*d^3 + d^4)*cos(f*x + e)^5 - (31*c^4 - 196*c^3*d + 154*c^2*d^2 - 4*c*d
^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 2*(8
1*c^4 - 252*c^3*d + 150*c^2*d^2 - 28*c*d^3 + d^4)*cos(f*x + e)^3 + 2*(79*c^
4 - 100*c^3*d + 74*c^2*d^2 - 4*c*d^3 - d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^
2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*
x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d + 12*c
*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x + e) +
 ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d - 17*c
*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*c^3
- 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(d*sin(f*x + e) + c)*sqrt(c/a) + (289*c^4 - 476*c^3*d + 230*c^2*d
^2 - 28*c*d^3 + d^4)*cos(f*x + e) + ((c^4 - 28*c^3*d + 70*c^2*d^2 - 28*c*d^
3 + d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 + 32*(c
^4 - 7*c^3*d + 7*c^2*d^2 - c*d^3)*cos(f*x + e)^3 - 2*(65*c^4 - 140*c^3*d +
```

```
38*c^2*d^2 - 12*c*d^3 + d^4)*cos(f*x + e)^2 - 32*(9*c^4 - 15*c^3*d + 7*c^2*
d^2 - c*d^3)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 -
 2*cos(f*x + e)^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)*sin(f*x + e) + cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*arc
tan(1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)
*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c*d - d^2)*cos(f*x + e)*sin(f*
x + e) + (c^2 - c*d)*cos(f*x + e))) + sqrt(-c/a)*arctan(-1/4*((c^2 - 6*c*d
+ d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-c/a)/((c^2*d - c*d^2
)*cos(f*x + e)^3 - (c^3 - 3*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*c^3 - c^2
*d + c*d^2)*cos(f*x + e))))/f]
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{c+d\sin{(e+fx)}}}{\sqrt{a(\sin{(e+fx)}+1)}\sin{(e+fx)}}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sin(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\sin{(fx+e)}+c}}{\sqrt{a\sin{(fx+e)}+a}\sin{(fx+e)}}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((c + d*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)), x
)
```

3.38 $\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

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Optimal result

Integrand size = 35, antiderivative size = 140

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*c^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e
))^(1/2))/f/a^(1/2)/c^(1/2)+arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^
(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3026, 2861, 214, 3022, 212}

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[c]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[
c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3022

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst
[Int[1/(1 - a*c*x^2), x], x, Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[b*c + a*d, 0]
```

Rule 3026

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*S
qrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-b/a, Int[1/(
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[1/a, Int[
Sqrt[a + b*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] ||
 NeQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a} - \int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1-acx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} + \frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 885 vs. $2(140) = 280$.

Time = 14.91 (sec) , antiderivative size = 885, normalized size of antiderivative = 6.32

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= \frac{\csc(e+fx)\left(\frac{\log\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{c}} - \frac{\sqrt{2}\log\left(1+\tan\left(\frac{1}{2}(e+f\right.\right.}{\sqrt{c-d}}\right.}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c+d\sin(e+fx)}\left(\frac{\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)}{2\sqrt{c}} - \frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{\sqrt{2}\sqrt{c-d}\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)} + \frac{\frac{1}{2}c\sec^2\left(\frac{1}{2}(e+f\right.}{}\right.}$$

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x]
```

```
[Out] (Csc[e + f*x]*(Log[Tan[(e + f*x)/2]]/Sqrt[c] - (Sqrt[2]*Log[1 + Tan[(e + f*
x)/2]])/Sqrt[c - d] + Log[d + Sqrt[2]*Sqrt[c]*Sqrt[(1 + Cos[e + f*x])^(-1)]
*Sqrt[c + d*Sin[e + f*x]] + c*Tan[(e + f*x)/2]]/Sqrt[c] - Log[c + Sqrt[2]*S
qrt[c]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + d*Tan[(e +
f*x)/2]]/Sqrt[c] + (Sqrt[2]*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x
])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]])/Sqrt[c - d]
))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*x]]*((Csc[(e + f*x)/2
]*Sec[(e + f*x)/2])/(2*Sqrt[c]) - Sec[(e + f*x)/2]^2/(Sqrt[2]*Sqrt[c - d]*(
1 + Tan[(e + f*x)/2])) + ((c*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c]*d*Cos[e + f*x
]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt
[c]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/
Sqrt[2])/(Sqrt[c]*(d + Sqrt[2]*Sqrt[c]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c
 + d*Sin[e + f*x]] + c*Tan[(e + f*x)/2])) - ((d*Sec[(e + f*x)/2]^2)/2 + (Sq
rt[c]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin
[e + f*x]]) + (Sqrt[c]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/Sqrt[2])/(Sqrt[c]*(c + Sqrt[2]*Sqrt[c]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + d*Tan[(e + f*x)/2])) + (Sqrt[2]*(((
```

```
-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
 + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^
(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(Sqrt[c - d]*(c - d + 2
*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c +
 d)*Tan[(e + f*x)/2])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(113) = 226.

Time = 1.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

method	result
default	$\frac{\sqrt{c+d\sin(fx+e)}\,\sqrt{2}\left(\ln\left(-\frac{-\sqrt{c}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}+c\cot(fx+e)-c\csc(fx+e)-d}{\sqrt{c}}\right)\sqrt{2c-2d}-\ln\left(-\frac{2\left(\sqrt{c}\,\sqrt{2}\,\sqrt{\frac{c+d\sin(fx+e)}{1+\cos(fx+e)}}\,\sin(fx+e)+c}{\cos(fx+e)-1}\right.\right.}{2f(1+\cos(f}$

```
[In] int(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*(c+d*sin(f*x+e))^(1/2)*2^(1/2)*(ln(-(-c^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(1+cos(f*x+e)))^(1/2)+c*cot(f*x+e)-c*csc(f*x+e)-d)/c^(1/2))*(2*c-2*d)^(1
/2)-ln(-2*(c^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)+c*sin(f*x+e)-cos(f*x+e)*d+d)/(cos(f*x+e)-1))*(2*c-2*d)^(1/2)+2*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-cos(f*x+e)*d-c+d)/(-cos(f*x+e)+1+sin(f*
x+e)))*c^(1/2))*(cos(f*x+e)+sin(f*x+e)+1)/(1+cos(f*x+e))/(a*(1+sin(f*x+e)))
^(1/2)/((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)/c^(1/2)/(2*c-2*d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(113) = 226.

Time = 0.71 (sec) , antiderivative size = 3005, normalized size of antiderivative = 21.46

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*c*log(((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 - (13*c^2 - 2
2*c*d - 3*d^2)*cos(f*x + e)^2 + 4*sqrt(2)*((c^2 - 4*c*d + 3*d^2)*cos(f*x +
e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e) + (4*c^2
- 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a*c - a*d) - 4*c^2 - 8*c*d
```

- 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f *x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4))/sqrt(a*c - a*d) + sqrt(a*c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*c os(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a *c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2* (81*a*c^4 - 252*a*c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^ 3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c ^2*d + 5*c*d^2)*cos(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18 *c^3 - 33*c^2*d + 12*c*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5 *c*d^2)*cos(f*x + e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51 *c^3 + 59*c^2*d - 17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos (f*x + e)^2 - (25*c^3 - 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a*c^4 - 476*a*c^3*d + 230*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a* c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a* c^2*d^2 - a*c*d^3)*cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^ 2 - 12*a*c*d^3 + a*d^4)*cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2 *d^2 - a*c*d^3)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^ 4 - 2*cos(f*x + e)^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^ 2 + 1)*sin(f*x + e) + cos(f*x + e) + 1)))/(a*c*f), 1/4*(sqrt(2)*a*c*log(((c ^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 + 4*sqrt(2)*((c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4* d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d* sin(f*x + e) + c)/sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c *d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(c os(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*si n(f*x + e) - 2*cos(f*x + e) - 4))/sqrt(a*c - a*d) + 2*sqrt(-a*c)*arctan(-1/ 4*((c^2 - 6*c*d + d^2)*cos(f*x + e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d) *sin(f*x + e))*sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) /((a*c^2*d - a*c*d^2)*cos(f*x + e)^3 - (a*c^3 - 3*a*c^2*d)*cos(f*x + e)*sin (f*x + e) + (2*a*c^3 - a*c^2*d + a*c*d^2)*cos(f*x + e))))/(a*c*f), -1/4*(2* sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))) - sqrt(a*c)*log(((a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^5 + a* c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (31*a*c^4 - 196*a*c^3*d + 154*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 2*(81*a*c^4 - 252*a* c^3*d + 150*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e)^3 + 2*(79*a*c^4 - 100*a*c^3*d + 74*a*c^2*d^2 - 4*a*c*d^3 - a*d^4)*cos(f*x + e)^2 - 8*((c^3 -

```
7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^4 - 2*(5*c^3 - 14*c^2*d + 5*c*d^2)*co
s(f*x + e)^3 + 51*c^3 - 59*c^2*d + 17*c*d^2 - d^3 - 2*(18*c^3 - 33*c^2*d +
12*c*d^2 - d^3)*cos(f*x + e)^2 + 2*(13*c^3 - 14*c^2*d + 5*c*d^2)*cos(f*x +
e) + ((c^3 - 7*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e)^3 - 51*c^3 + 59*c^2*d -
17*c*d^2 + d^3 + (11*c^3 - 35*c^2*d + 17*c*d^2 - d^3)*cos(f*x + e)^2 - (25*
c^3 - 31*c^2*d + 7*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (289*a*c^4 - 476*a*c^3*d + 2
30*a*c^2*d^2 - 28*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*
c^2*d^2 + 4*a*c*d^3 + a*d^4 + (a*c^4 - 28*a*c^3*d + 70*a*c^2*d^2 - 28*a*c*d
^3 + a*d^4)*cos(f*x + e)^4 + 32*(a*c^4 - 7*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)
*cos(f*x + e)^3 - 2*(65*a*c^4 - 140*a*c^3*d + 38*a*c^2*d^2 - 12*a*c*d^3 + a
*d^4)*cos(f*x + e)^2 - 32*(9*a*c^4 - 15*a*c^3*d + 7*a*c^2*d^2 - a*c*d^3)*co
s(f*x + e))*sin(f*x + e))/(cos(f*x + e)^5 + cos(f*x + e)^4 - 2*cos(f*x + e)
^3 - 2*cos(f*x + e)^2 + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sin(f*x + e
) + cos(f*x + e) + 1)))/(a*c*f), -1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*arc
tan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d
)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e
) + c*cos(f*x + e))) - sqrt(-a*c)*arctan(-1/4*((c^2 - 6*c*d + d^2)*cos(f*x
+ e)^2 - 9*c^2 + 6*c*d - d^2 + 8*(c^2 - c*d)*sin(f*x + e))*sqrt(-a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((a*c^2*d - a*c*d^2)*cos(f*x +
 e)^3 - (a*c^3 - 3*a*c^2*d)*cos(f*x + e)*sin(f*x + e) + (2*a*c^3 - a*c^2*d
+ a*c*d^2)*cos(f*x + e))))/(a*c*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))*sin(e + f*x
)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),
 x)
```

3.39 $\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} \, dx$

Optimal result 270
Rubi [A] (verified) 270
Mathematica [A] (verified) 272
Maple [A] (verified) 273
Fricas [B] (verification not implemented) 273
Sympy [F(-1)] 275
Maxima [F(-2)] 275
Giac [A] (verification not implemented) 276
Mupad [B] (verification not implemented) 276

Optimal result

Integrand size = 33, antiderivative size = 181

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} \, dx = -\frac{2a(a^2c-2b^2c+abd)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2f} + \frac{2c^2\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\sqrt{c^2-d^2}f} + \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))}$$

```
[Out] -2*a*(a^2*c+a*b*d-2*b^2*c)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))
/(a^2-b^2)^(3/2)/(-a*d+b*c)^2/f+a^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*
sin(f*x+e))+2*c^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*
c)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3135, 3080, 2739, 632, 210}

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))} \, dx = -\frac{2a(a^2c+abd-2b^2c)\arctan\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{3/2}(bc-ad)^2} + \frac{a^2\cos(e+fx)}{f(a^2-b^2)(bc-ad)(a+b\sin(e+fx))} + \frac{2c^2\arctan\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(bc-ad)^2}$$

```
[In] Int[Sin[e + f*x]^2/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*a*(a^2*c - 2*b^2*c + a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2*f) + (2*c^2*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]*f) + (a^2*Cos[e +
 f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x]))
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 2739</u>

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

<u>Rule 3080</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

<u>Rule 3135</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
```

```
0])))
```

Rubi steps

$$\text{integral} = \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{-abc-(a^2c-b^2c+abd)\sin(e+fx)}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx}{(a^2-b^2)(bc-ad)}$$

$$= \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} + \frac{c^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{(a(a^2c-2b^2c+abd))\int \frac{1}{a+b\sin(e+fx)}\,dx}{(a^2-b^2)(bc-ad)^2}$$

$$= \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)^2 f} - \frac{(2a(a^2c-2b^2c+abd))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(a^2-b^2)(bc-ad)^2 f}$$

$$= \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))} - \frac{(4c^2)\,\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)^2 f} + \frac{(4a(a^2c-2b^2c+abd))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(a^2-b^2)(bc-ad)^2 f}$$

$$= -\frac{2a(a^2c-2b^2c+abd)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2 f} + \frac{2c^2\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\sqrt{c^2-d^2}\,f} + \frac{a^2\cos(e+fx)}{(a^2-b^2)(bc-ad)f(a+b\sin(e+fx))}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{-\frac{2a(a^2c-2b^2c+abd)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}(bc-ad)^2} + \frac{2c^2\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)^2\sqrt{c^2-d^2}} - \frac{a^2\cos(e+fx)}{(a-b)(a+b)(-bc+ad)(a+b\sin(e+fx))}}{f}$$

```
[In] Integrate[Sin[e + f*x]^2/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((-2*a*(a^2*c - 2*b^2*c + a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
 b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2) + (2*c^2*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]) - (a^2*Cos[e + f*
x])/((a - b)*(a + b)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])))/f
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{2a\left(\frac{\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}+\frac{a(ad-bc)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(a^2c+abd-2b^2c\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{a^2d^2-2abcd+b^2c^2}+\frac{8c^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right.}{\left(4a^2d^2-8abcd+4b^2c^2\right)\sqrt{c^2-d}}}{f}$
default	$\frac{-\frac{2a\left(\frac{\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}+\frac{a(ad-bc)}{a^2-b^2}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{\left(a^2c+abd-2b^2c\right)\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}\right)}{a^2d^2-2abcd+b^2c^2}+\frac{8c^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right.}{\left(4a^2d^2-8abcd+4b^2c^2\right)\sqrt{c^2-d}}}{f}$
risch	$\frac{2ia^2\left(ib+a\,e^{i(fx+e)}\right)}{b(a^2-b^2)(ad-bc)f\left(-ie^{2i(fx+e)}b+2a\,e^{i(fx+e)}+ib\right)}+\frac{c^2\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-c^2+d^2}+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)}{\sqrt{-c^2+d^2}\,(ad-bc)^2f}-\frac{c^2\ln\left(e^{i(fx+e)}+\frac{ic\sqrt{-}}{\sqrt{}}\right.}{\sqrt{-c^2+d^2}\,(ad}$

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)*((b*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1
/2*e)+a*(a*d-b*c)/(a^2-b^2))/(tan(1/2*f*x+1/2*e)^2*a+2*b*tan(1/2*f*x+1/2*e)
+a)+(a^2*c+a*b*d-2*b^2*c)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e
)+2*b)/(a^2-b^2)^(1/2)))+8*c^2/(4*a^2*d^2-8*a*b*c*d+4*b^2*c^2)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(171) = 342$.

Time = 65.07 (sec) , antiderivative size = 2837, normalized size of antiderivative = 15.67

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 - 2*a^2*b^2)*
c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 - (a^3*b - 2*a
*b^3)*c*d^2)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)
^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*co
s(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^
2 - b^2)) - ((a^4*b - 2*a^2*b^3 + b^5)*c^2*sin(f*x + e) + (a^5 - 2*a^3*b^2
+ a*b^4)*c^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*s
qrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2
*((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3)*c*d^2 +
 (a^5 - a^3*b^2)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(
a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*
b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b -
2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2
+ 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*
f), -1/2*(2*((a^4*b - 2*a^2*b^3 + b^5)*c^2*sin(f*x + e) + (a^5 - 2*a^3*b^2
+ a*b^4)*c^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)
*cos(f*x + e))) - (a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 -
 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 -
(a^3*b - 2*a*b^3)*c*d^2)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*
cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x
 + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f
*x + e) - a^2 - b^2)) - 2*((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d -
(a^4*b - a^2*b^3)*c*d^2 + (a^5 - a^3*b^2)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2
*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 -
(a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 +
 a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*
a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2
*a^5*b^2 + a^3*b^4)*d^4)*f), 1/2*(2*(a^3*b*c^2*d - a^3*b*d^3 + (a^4 - 2*a^2
*b^2)*c^3 - (a^4 - 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^2*b^2*d^3 + (a^3*b
 - 2*a*b^3)*c^3 - (a^3*b - 2*a*b^3)*c*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*ar
ctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((a^4*b - 2*a^
2*b^3 + b^5)*c^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*c^2)*sqrt(-c^2 +
d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2
*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(
f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((a^4*b - a^2*b^3)*c^3 -
(a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3)*c*d^2 + (a^5 - a^3*b^2)*d^3)*cos(
f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^
6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a
^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) +
 ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d
 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a
^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), ((a^3*b*c^2*d - a^3*b*
```

```
d^3 + (a^4 - 2*a^2*b^2)*c^3 - (a^4 - 2*a^2*b^2)*c*d^2 + (a^2*b^2*c^2*d - a^
2*b^2*d^3 + (a^3*b - 2*a*b^3)*c^3 - (a^3*b - 2*a*b^3)*c*d^2)*sin(f*x + e))*
sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))
) - ((a^4*b - 2*a^2*b^3 + b^5)*c^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)
*c^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + ((a^4*b - a^2*b^3)*c^3 - (a^5 - a^3*b^2)*c^2*d - (a^4*b - a^2*b^3
)*c*d^2 + (a^5 - a^3*b^2)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*
c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^
5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b
^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*
(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b
^4)*d^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.61

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{2\left(\frac{\left(\pi\left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)c^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{c^2-d^2}} - \frac{(a^3c-2ab^2c+a^2bd)\left(\pi\left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^2c^2-b^4c^2-2a^3bcd+2ab^3cd+a^4d^2-a^2b^2d^2)\sqrt{a^2-b^2}} + \frac{}{(a}\right.}{f}$$

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*c^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2 - d^
2)) - (a^3*c - 2*a*b^2*c + a^2*b*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^2*c^2 - b
^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d + a^4*d^2 - a^2*b^2*d^2)*sqrt(a^2 - b^2)
) + (a*b*tan(1/2*f*x + 1/2*e) + a^2)/((a^2*b*c - b^3*c - a^3*d + a*b^2*d)*(
a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a))/f
```

Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 23933, normalized size of antiderivative = 132.23

$$\int \frac{\sin^2(e+fx)}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

```
[In] int(sin(e + f*x)^2/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] - ((2*a^2)/((a^2 - b^2)*(a*d - b*c)) + (2*a*b*tan(e/2 + (f*x)/2))/((a^2 - b
^2)*(a*d - b*c)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2))
 - (c^2*atan(((c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^
6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d
^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^
4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*
a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b
^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^7*b
*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^
3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c
^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c
^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^
```

7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3
 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d +
3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (c^2*(d^2 - c^2)^(1/2)*((32*(a*b^9*c^7 - a
^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6
*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3
- 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^
4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*
b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^
3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4
*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2
 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4
 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d
+ 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2
+ 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5
*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 1
2*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^
5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 +
a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3
*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (c^2*(d^2 - c^2
)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2
*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*
b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a
^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6
 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^
5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*
b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11
*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 -
 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^
5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*
b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2
*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4
*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 1
0*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*
d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4
*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^
2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^
7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d
^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10
*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a
^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*
d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a
^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))
/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d
))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b
*c^3*d) + (c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^6*b^

2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (c^2*(d^2 - c^2)^(1/2)*((32*(a*b^9*c^7 - a^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^

5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))/((64*(a^5*b*c^5 - 2*a^3*b^3*c^5 + a^4*b^2*c^4*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^6*c^5 + 4*a^2*b^4*c^5 - 6*a^4*b^2*c^5 - 6*a^3*b^3*c^4*d + 2*a^4*b^2*c^3*d^2 + 4*a^5*b*c^4*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (c^2*(d^2 - c^2)^(1/2)*((32*(a*b^9*c^7 - a^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c

^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*
d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5
*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b
^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*
a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^
2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^
7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3
*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*
d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x
)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b
^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8
*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4
*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2
*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^
5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*
b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^
6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8
*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2
 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b
^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^
3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c
*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*
b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2
*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3
 + 2*a*b*c^3*d) - (c^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 -
 a^6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^
3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2
*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 -
 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^
5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^
7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8
*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^
2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^
4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/
(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*
d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d
 + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (c^2*(d^2 - c^2)^(1/2)*((32*(a*b^9*c^7
- a^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*
c^6*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d
^3 - 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4
*c^4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a
^8*b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3
*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*
a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(
e/2 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*

d^4 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6
*d + 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d
^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*
b^5*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2
- 12*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2
*c^5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3
+ a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*
b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (c^2*(d^2 -
c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6
+ 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a
^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 +
3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*
d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5
*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a
^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a
^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^
3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3
*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3
*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8
+ 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d
+ 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d
+ 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c
^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*
a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6
*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140
*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^
5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a
^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3
- a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c
^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))
/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d
)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^
3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b
*c^3*d)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c
^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)) - (a*atan(((a*(-(a + b)^3*(a - b)^3)^(
1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*
b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b
^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a
^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 -
6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6
*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d
+ 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4
*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b
*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13
*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3

- a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c ^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^9*c^7 - a^3*b^7*c^7 + a^10*c^2*d ^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6*d - a^7*b^3*c*d^6 - 4* a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b^5*c^3*d^4 + 3 *a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 13*a^7*b^3*c^3* d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d^3 + a^9*b*c*d ^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5 *b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2* c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(4*a^2*b^8* c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3*b^7*c^6*d + 2 6*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b^2*c*d^6 + 2*a ^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5 - 48*a^5*b^5*c ^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3*c^2*d^5 + 36* a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d)) /(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2 *d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2* d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (a*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5* c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c ^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5 *b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4* d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b ^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c ^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^ 4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7 *a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b ^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8* b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a ^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d ^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5 *b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d ^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^ 8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^ 4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d ^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a *b^6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8* c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4

*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^
3*c*d))*(a^2*c - 2*b^2*c + a*b*d)*1i)/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 -
3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 +
2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) + (a*(-(a + b)^
3*(a - b)^3)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^6*b^2*c^6 + a^8*c^
4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4
*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^4*d^2))/(a^7*d^3 -
b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^
2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6
*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6
- 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*
c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d + 4*a^7*b*c^2
*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 - 10*a^5*b^
3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 +
2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2
- 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*
a^6*b*c*d^2) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^9*c^7 - a^3*b^7*c^
7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6*d - a^7*
b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b
^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 1
3*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d
^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b
^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^
2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/
2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3
*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b
^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*
b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5
- 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3
*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2
*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d
^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 +
3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (a*((32*(2*a^4*b^8*c^8 -
a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^
7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5
- 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^
3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*
b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 1
0*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*
d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3
+ 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*
d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d -
3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a
^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*
d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c

^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*
c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41
*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2
*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130
*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^
6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*
a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^
5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*
a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b
^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^
2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2
+ a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d -
2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d))/(
a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d
^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*
c*d + 6*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d)*1i)/(a^8*d^2 - b^8*c^2 + 3*
a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3
*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))/
((64*(a^5*b*c^5 - 2*a^3*b^3*c^5 + a^4*b^2*c^4*d))/(a^7*d^3 - b^7*c^3 + 2*a^
2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6
*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*
b*c*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^6*c^5 + 4*a^2*b^4*c^5 - 6*a^4*b^2*c^
5 - 6*a^3*b^3*c^4*d + 2*a^4*b^2*c^3*d^2 + 4*a^5*b*c^4*d))/(a^7*d^3 - b^7*c^
3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c
*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d
- 3*a^6*b*c*d^2) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^4*b^4*c^6 - a^
2*b^6*c^6 - a^6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d +
2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4
- 6*a^6*b^2*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^
3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c
*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*
x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*
c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5
+ 10*a^6*b^2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2
+ 16*a^4*b^4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2
*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 -
2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^
5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (a*(-(a + b)^3*(a - b)^3)^(1
/2)*((32*(a*b^9*c^7 - a^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*
b^6*c^6*d + a^6*b^4*c^6*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d
^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*
c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a
^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a
^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 -
6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6

*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (a*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^4*b^4*c^6 - a^2*b^6*c^6 - a^6*b^2*c^6 + a^8*c^4*d^2 - 3*a^3*b^5*c^5*d + 2*a^5*b^3*c^5*d + 2*a^7*b*c^3*d^3 + 8*a^4*b^4*c^4*d^2 - 5*a^5*b^3*c^3*d^3 + a^6*b^2*c^2*d^4 - 6*a^6*b^2*c^4*d^2))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*ta n(e/2 + (f*x)/2)*(2*a^7*b*c^6 - 2*a*b^7*c^6 - 2*a^8*c^5*d + 9*a^3*b^5*c^6 - 8*a^5*b^3*c^6 + 2*a^8*c^3*d^3 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c^5*d + 2*a^6*b^2*c*d^5 + 10*a^6*b^2*c^5*d + 4*a^7*b*c^2*d^4 - 4*a^7*b*c^4*d^2 - 8*a^3*b^5*c^4*d^2 + 16*a^4*b^4*c^3*d^3 - 10*a^5*b^3*c^2*d^4 + 13*a^5*b^3*c^4*d^2 - 13*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^9*c^7 - a^3*b^7*c^7 + a^10*c^2*d^5 - 3*a^2*b^8*c^6*d + 2*a^4*b^6*c^6*d + a^6*b^4*c^6*d - a^7*b^3*c*d^6 - 4*a^9*b*c^3*d^4 + a^3*b^7*c^5*d^2 + 6*a^4*b^6*c^4*d^3 - 9*a^5*b^5*c^3*d^4 + 3*a^5*b^5*c^5*d^2 + 5*a^6*b^4*c^2*d^5 - 12*a^6*b^4*c^4*d^3 + 13*a^7*b^3*c^3*d^4 - 4*a^7*b^3*c^5*d^2 - 6*a^8*b^2*c^2*d^5 + 6*a^8*b^2*c^4*d^3 + a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(4*a^2*b^8*c^7 - 6*a^4*b^6*c^7 + 2*a^6*b^4*c^7 + 2*a^10*c^3*d^4 - 20*a^3*b^7*c^6*d + 26*a^5*b^5*c^6*d - 2*a^6*b^4*c*d^6 - 8*a^7*b^3*c^6*d + 2*a^8*b^2*c*d^6 + 2*a^9*b*c^2*d^5 - 8*a^9*b*c^4*d^3 - 10*a^2*b^8*c^5*d^2 + 20*a^3*b^7*c^4*d^3 - 20*a^4*b^6*c^3*d^4 + 42*a^4*b^6*c^5*d^2 + 10*a^5*b^5*c^2*d^5 - 48*a^5*b^5*c^4*d^3 + 32*a^6*b^4*c^3*d^4 - 44*a^6*b^4*c^5*d^2 - 12*a^7*b^3*c^2*d^5 + 36*a^7*b^3*c^4*d^3 - 14*a^8*b^2*c^3*d^4 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (a*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6

```
*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2
 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*
a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6
*a^5*b^3*c*d))*(a^2*c - 2*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^
2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d
^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(a^2*c - 2
*b^2*c + a*b*d))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b
^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*
b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^2*
c - 2*b^2*c + a*b*d)*2i)/(f*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*
c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c
*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))
```

3.40 $\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal result 288
Rubi [A] (verified) 288
Mathematica [C] (verified) 290
Maple [A] (verified) 290
Fricas [F(-1)] 291
Sympy [F] 291
Maxima [F] 291
Giac [F] 292
Mupad [F(-1)] 292

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{2c\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{a(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*c*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
Pi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(
c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)+2*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f
*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x)
,2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/(a+b)/
f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{3014, 2886, 2884\}$

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{2c\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2d}{c+d}\right)}{af(a+b)\sqrt{c+d\sin(e+fx)}}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*c*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f*x]]) - (2*(b*c - a*d)*EllipticPi[(
2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(a*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3014

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*((c
_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[a/c, Int[1/(Sin[e +
 f*x]*Sqrt[a + b*Sin[e + f*x]]), x], x] + Dist[(b*c - a*d)/c, Int[1/(Sqrt[a
 + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{c\int \frac{\csc(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a} + \frac{(-bc+ad)\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(c\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{\csc(e+fx)}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sin(e+fx)}} + \frac{\left((-bc+ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2c\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{a(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.02 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{2i\left(\text{EllipticPi}\left(\frac{c+d}{c},i\text{arcsinh}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right),\frac{c+d}{c-d}\right)-\text{EllipticPi}\left(\frac{b(c+d)}{bc-ad},i\text{arcsinh}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c\right.\right.\right.}{a\sqrt{-\frac{1}{c+d}}f}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((2*I)*(EllipticPi[(c + d)/c, I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] - EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSin
h[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sec[e +
f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(
-c + d)])/(a*Sqrt[-(c + d)^(-1)]*f)
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

method	result
default	$-\frac{2\left(\Pi\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\frac{c-d}{c},\sqrt{\frac{c-d}{c+d}}\right)-\Pi\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},-\frac{(c-d)b}{ad-bc},\sqrt{\frac{c-d}{c+d}}\right)\right)\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{\frac{c+d\sin(fx+e}{c-d}}}{a\cos(fx+e)\sqrt{c+d\sin(fx+e)}\,f}$

```
[In] int((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*(EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(c-d)/c,((c-d)/(c+d))^(1/2))-
EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^
(1/2)))*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(c-d)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\left(a+b\sin\left(e+fx\right)\right)\sin\left(e+fx\right)}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/((a + b*sin(e + f*x))*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/((b*sin(f*x + e) + a)*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)\sin(fx+e)}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/((b*sin(f*x + e) + a)*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{c+d\sin(e+fx)}}{\sin(e+fx)\,(a+b\sin(e+fx))}\,dx$$

```
[In] int((c + d*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

3.41 $\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result . . . 293
Rubi [A] (verified) . . . 293
Mathematica [C] (verified) . . . 295
Maple [A] (verified) . . . 295
Fricas [F(-1)] . . . 296
Sympy [F] . . . 296
Maxima [F] . . . 296
Giac [F] . . . 297
Mupad [F(-1)] . . . 297

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{a(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)+2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),
2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/(a+b)/f/(c+d*sin(
f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{3020, 2886, 2884\}$

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}} - \frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),\frac{2d}{c+d}\right)}{af(a+b)\sqrt{c+d\sin(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f*x]]) - (2*b*EllipticPi[(2*b)/(a + b)
, (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a
*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3020

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c, Int[1/(Sin[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/c, Int[1/(Sqrt[a + b*Sin
[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\csc(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a} - \frac{b\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\int \frac{\csc(e+fx)}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sin(e+fx)}} - \frac{\left(b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2\,\text{EllipticPi}\left(2,\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{af\sqrt{c+d\sin(e+fx)}} - \frac{2b\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{a(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.59 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.39

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2i\left((-bc+ad)\,\text{EllipticPi}\left(\frac{c+d}{c},i\text{arcsinh}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right),\frac{c+d}{c-d}\right)+bc\,\text{EllipticPi}\left(\frac{b(c+d)}{bc-ad},i\text{arc}\right.\right.}{ac\sqrt{-\frac{1}{c+d}}(bc-a}$$

```
[In] Integrate[Csc[e + f*x]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((-2*I)*((-(b*c) + a*d)*EllipticPi[(c + d)/c, I*ArcSinh[Sqrt[-(c + d)^(-1)]
*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + b*c*EllipticPi[(b*(c + d))/(
b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d
*(1 + Sin[e + f*x]))/(c - d))])/(a*c*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*f)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.74

method	result
default	$-\frac{2(c-d)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\left(\Pi\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\frac{c-d}{c},\sqrt{\frac{c-d}{c+d}}\right)ad-\Pi\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}},\frac{c-d}{c},\sqrt{\right.\right.}{ac(ad-bc)\cos(fx+e)\sqrt{c+d\sin(fx+e)}\,f}$

```
[In] int(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(c-
d)/c,((c-d)/(c+d))^(1/2))*a*d-EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(c-
```

```
d)/c,((c-d)/(c+d))^(1/2))*b*c+b*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-
(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*c)/a/c/(a*d-b*c)/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\sin(e+fx)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{(b\sin(fx+e)+a)\sqrt{d\sin(fx+e)+c}\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$$
$$= \int \frac{1}{(b\sin(fx+e)+a)\sqrt{d\sin(fx+e)+c}\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm
="giac")
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$$
$$= \int \frac{1}{\sin(e+fx)\,(a+b\sin(e+fx))\,\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

3.42 $\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

Optimal result 298
Rubi [A] (verified) 298
Mathematica [C] (warning: unable to verify) 300
Maple [C] (warning: unable to verify) 300
Fricas [F(-1)] 301
Sympy [F] 301
Maxima [F] 301
Giac [F] 301
Mupad [F(-1)] 302

Optimal result

Integrand size = 39, antiderivative size = 254

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a+b}\sqrt{g}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{df} - \frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] 2*EllipticPi(g^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(g*sin(f*x+e))^(1/2
),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*g^(1/2)*(a*(1-csc(f*x+e))/(a+b)
)^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/d/f-2*(-a*d+b*c)*Elliptic
Pi(1/2*(1-csc(f*x+e))^(1/2)*2^(1/2),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*(-co
t(f*x+e)^2)^(1/2)*((b+a*csc(f*x+e))/(a+b))^(1/2)*(g*sin(f*x+e))^(1/2)*tan(f
*x+e)/d/(c+d)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

= {3008, 2888, 3016}

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$$

$$= \frac{2\sqrt{g}\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)}{df} - \frac{2(bc-ad)\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{a\csc(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+}}{\sqrt{2}}\right.\right.}{df(c+d)\sqrt{a+b\sin(e+fx)}}$$

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Sqrt[g]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Cs
c[e + f*x]))/(a - b)]*EllipticPi[(a + b)/b, ArcSin[(Sqrt[g]*Sqrt[a + b*Sin[
e + f*x]])/(Sqrt[a + b]*Sqrt[g*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e +
 f*x])/(d*f) - (2*(b*c - a*d)*Sqrt[-Cot[e + f*x]^2]*Sqrt[(b + a*Csc[e + f*x
])/(a + b)]*EllipticPi[(2*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]
], (2*a)/(a + b)]*Sqrt[g*Sin[e + f*x]]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a +
b*Sin[e + f*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 3008

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]])/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[b/d, In
t[Sqrt[g*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/
d, Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
```

d^2, 0]

Rubi steps

$$\text{integral} = \frac{b\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\sqrt{g}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+}{df}$$

$$-\frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e}{d(c+d)f\sqrt{a+b\sin(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.90 (sec) , antiderivative size = 23019, normalized size of antiderivative = 90.63

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(c + d*Sin[e + f*
x]),x]
[Out] Result too large to show
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 5478, normalized size of antiderivative = 21.57

method	result	size
default	Expression too large to display	5478

```
[In] int((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_
RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{g\sin{(e+fx)}}\sqrt{a+b\sin{(e+fx)}}}{c+d\sin{(e+fx)}}\,dx$$

```
[In] integrate((g*sin(f*x+e))**(1/2)*(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}+a}\sqrt{g\sin{(fx+e)}}}{d\sin{(fx+e)}+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(d*sin(f*x + e) + c
), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}+a}\sqrt{g\sin{(fx+e)}}}{d\sin{(fx+e)}+c}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/(d*sin(f*x + e) + c
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)
),x)
```

```
[Out] int(((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)
), x)
```

3.43 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal result 303
Rubi [A] (verified) 303
Mathematica [F] 305
Maple [B] (warning: unable to verify) 305
Fricas [F(-1)] 307
Sympy [F] 307
Maxima [F] 307
Giac [F] 308
Mupad [F(-1)] 308

Optimal result

Integrand size = 39, antiderivative size = 250

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{cf\sqrt{g}} + \frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\,t}{c(c+d)fg\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*EllipticF(g^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(g*sin(f*x+e))^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+cs
c(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/c/f/g^(1/2)+2*(-a*d+b*c)*EllipticPi(1/2*(
1-csc(f*x+e))^(1/2)*2^(1/2),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*(-cot(f*x+e)
^2)^(1/2)*((b+a*csc(f*x+e))/(a+b))^(1/2)*(g*sin(f*x+e))^(1/2)*tan(f*x+e)/c/
(c+d)/f/g/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{3012, 2895, 3016\}$

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$

$$= \frac{2(bc-ad)\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{a\csc(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right.\right.}{cfg(c+d)\sqrt{a+b\sin(e+fx)}}$$

$$-\frac{2\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)}{cf\sqrt{g}}$$

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f
*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[g]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
+ b]*Sqrt[g*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(c*f*Sqrt[g]
) + (2*(b*c - a*d)*Sqrt[-Cot[e + f*x]^2]*Sqrt[(b + a*Csc[e + f*x])/(a + b)]
*EllipticPi[(2*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*a)/(a
 + b)]*Sqrt[g*Sin[e + f*x]]*Tan[e + f*x])/(c*(c + d)*f*g*Sqrt[a + b*Sin[e +
 f*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3012

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(g_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[a/c, In
t[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] + Dist[(b*c - a
*d)/(c*g), Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx}{c} + \frac{(bc-ad)\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{cg}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{cf\sqrt{g}} + \frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e}}{c(c+d)fg\sqrt{a+b\sin(e+fx)}}$$

Mathematica [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))),x]
[Out] Integrate[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[g*Sin[e + f*x]]*(c + d*Sin[e + f*x
])), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2883 vs. 2(231) = 462.

Time = 2.50 (sec) , antiderivative size = 2884, normalized size of antiderivative = 11.54

method	result	size
default	Expression too large to display	2884

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
[Out] 1/f*(a*d-b*c)*(2*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b
^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b
^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((
b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b+2*(-a^2+b^2)^(1/2)*(-c^2+d^2
)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2
+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^
```

```
2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)
)*b-4*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(
a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+
b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b+(-a^2+b^2)^(1/2)*EllipticPi((1/(b+(-
a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-
a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/
2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*c-2*(-a^2+b^2)^(1/2)*El
lipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2
)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a
*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b*d-(-a^
2+b^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+
(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c
^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*a*c+2*(-a^2+b^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x
+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2
+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(
-a^2+b^2)^(1/2))^(1/2))*b*d-(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1
/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1
/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^
2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2+2*(-c^2+d^2)^(1/2)*EllipticPi((1
/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2)
,(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2
*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2-(-c^2+d^2)^(1/2
)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^
(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/
2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2+
2*(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f
*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)
-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(
1/2))^(1/2))*b^2+2*(-c^2+d^2)^(1/2)*EllipticF((1/(b+(-a^2+b^2)^(1/2))*(a*cs
c(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)
^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2-4*(-c^2+d^2)^(1/2)*EllipticF((1/(b+(-a
^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),1/2*2^
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2+EllipticPi((1/(b+(
-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(
-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*d+EllipticPi((1/(b+(
-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(
-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*c-2*EllipticPi((1/(b
+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b
+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*d-EllipticPi((1/(b
+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b
+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^
```

```
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*d-EllipticPi((1/(b
+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b
+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*c+2*EllipticPi((1/
(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),
(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*d)*((-csc(f*x+e)
+cot(f*x+e))*a/(b+(-a^2+b^2)^(1/2)))^(1/2)*((a*cot(f*x+e)-a*csc(f*x+e)+(-a^
2+b^2)^(1/2)-b)/(-a^2+b^2)^(1/2))^(1/2)*(1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+
e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2)*2^(1/2)*(1+cos(f*x+e))/(a+b*sin(
f*x+e))^(1/2)/(g*sin(f*x+e))^(1/2)/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a
*d+b*c)/(-c^2+d^2)^(1/2)/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{a+b\sin{(e+fx)}}}{\sqrt{g\sin{(e+fx)}}\,(c+d\sin{(e+fx)})}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))/(sqrt(g*sin(e + f*x))*(c + d*sin(e + f*x)
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}+a}}{(d\sin{(fx+e)}+c)\sqrt{g\sin{(fx+e)}}}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/((d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e
))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{g\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int((a + b*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
), x)
```

3.44 $\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal result 309
Rubi [A] (verified) 309
Mathematica [F] 310
Maple [B] (warning: unable to verify) 310
Fricas [F(-1)] 312
Sympy [F] 312
Maxima [F] 312
Giac [F] 313
Mupad [F(-1)] 313

Optimal result

Integrand size = 39, antiderivative size = 114

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{2\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\tan(e+fx)}{(c+d)f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] 2*EllipticPi(1/2*(1-csc(f*x+e))^(1/2)*2^(1/2),2*c/(c+d),2^(1/2)*(a/(a+b))^(
1/2))*(-cot(f*x+e)^2)^(1/2)*((b+a*csc(f*x+e))/(a+b))^(1/2)*(g*sin(f*x+e))^(
1/2)*tan(f*x+e)/(c+d)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {3016}

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{2\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{a\csc(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sin(e+fx)}}$$

```
[In] Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[-Cot[e + f*x]^2]*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[(2*c
)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*a)/(a + b)]*Sqrt[g*Si
n[e + f*x]]*Tan[e + f*x])/((c + d)*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

Rubi steps

integral

$$= \frac{2\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d}, \arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right), \frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\tan(e+fx)}{(c+d)f\sqrt{a+b\sin(e+fx)}}$$

Mathematica [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] Integrate[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x
])), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2582 vs. $2(107) = 214$.

Time = 2.80 (sec) , antiderivative size = 2583, normalized size of antiderivative = 22.66

method	result	size
default	Expression too large to display	2583

```
[In] int((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/f*(EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b
^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)
^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*
a^2*d-(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*c
```

ot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(
1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^
2)^(1/2))^(1/2))*a^2+EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot
(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/
2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)
^(1/2))^(1/2))*a*b*c+(-a^2+b^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a
*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/
(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^
(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*c-2*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a
*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/
(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^
(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*d+2*(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(
-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(
-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2-2*(-a^2+b^2)^(1/2)*E
llipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/
2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-
a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b*d+2*(
-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(
f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-
a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2)
)/(-a^2+b^2)^(1/2))^(1/2))*b-EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+
e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+
b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-
a^2+b^2)^(1/2))^(1/2))*a^2*d-(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(
1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(
1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a
^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2-EllipticPi((1/(b+(-a^2+b^2)^(1/
2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/
2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2
+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*c-(-a^2+b^2)^(1/2)*EllipticPi((1/
(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),
(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*c+2*EllipticPi((1/
(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),
(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*
2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*d+2*(-c^2+d^2)^(
1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^
2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^
(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b
^2+2*(-a^2+b^2)^(1/2)*EllipticPi((1/(b+(-a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*co
t(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1
/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2
)^(1/2))^(1/2))*b*d+2*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)*EllipticPi((1/(b+(-
a^2+b^2)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2),(b+(-

```
a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/
2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b)*(g*sin(f*x+e))^(1/2)*2
^(1/2)*((-csc(f*x+e)+cot(f*x+e))*a/(b+(-a^2+b^2)^(1/2)))^(1/2)*((a*cot(f*x+
e)-a*csc(f*x+e)+(-a^2+b^2)^(1/2)-b)/(-a^2+b^2)^(1/2))^(1/2)*(1/(b+(-a^2+b^2
)^(1/2))*(a*csc(f*x+e)-a*cot(f*x+e)+(-a^2+b^2)^(1/2)+b))^(1/2)/(a+b*sin(f*x
+e))^(1/2)*(cot(f*x+e)+csc(f*x+e))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)
-a*d+b*c)/(-c^2+d^2)^(1/2)/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(e+fx)}}}{\sqrt{a+b\sin{(e+fx)}}\,(c+d\sin{(e+fx)})}\,dx$$

```
[In] integrate((g*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x)
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin{(fx+e)}}}{\sqrt{b\sin{(fx+e)}+a}(d\sin{(fx+e)}+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\sin(fx+e)}}{\sqrt{b\sin(fx+e)+a}(d\sin(fx+e)+c)}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))/(a+b*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{\sqrt{g\,\sin(e+f\,x)}}{\sqrt{a+b\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))}\,dx$$

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))
), x)
```

3.45 $\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal result 314
Rubi [A] (verified) 314
Mathematica [B] (warning: unable to verify) 316
Maple [B] (warning: unable to verify) 316
Fricas [F] 318
Sympy [F] 319
Maxima [F] 319
Giac [F] 319
Mupad [F(-1)] 320

Optimal result

Integrand size = 39, antiderivative size = 246

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{acf\sqrt{g}} - \frac{2d\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\tan(e+f}{c(c+d)fg\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*EllipticF(g^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(g*sin(f*x+e))^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+cs
c(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a/c/f/g^(1/2)-2*d*EllipticPi(1/2*(1-csc(f
*x+e))^(1/2)*2^(1/2),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*(-cot(f*x+e)^2)^(1/
2)*((b+a*csc(f*x+e))/(a+b))^(1/2)*(g*sin(f*x+e))^(1/2)*tan(f*x+e)/c/(c+d)/f
/g/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used

= {3018, 2895, 3016}

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{2d\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{a\csc(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2}{a-}\right.}{cfg(c+d)\sqrt{a+b\sin(e+fx)}} - \frac{2\sqrt{a+b}\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)}{acf\sqrt{g}}$$

```
[In] Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])),
x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f
*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[g]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
+ b]*Sqrt[g*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(a*c*f*Sqrt[
g]) - (2*d*Sqrt[-Cot[e + f*x]^2]*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*Ellipti
cPi[(2*c)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*a)/(a + b)]*S
qrt[g*Sin[e + f*x]]*Tan[e + f*x])/(c*(c + d)*f*g*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

Rule 3018

```
Int[1/(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[1/c,
Int[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]), x], x] - Dist[d/(c*g
), Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
```

```
- b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx}{c} - \frac{d\int \frac{\sqrt{g\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx}{cg}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{g}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{g\sin(e+fx)}}\right),-\frac{a+b}{a-b}\right)\tan(e+fx)}{acf\sqrt{g}} - \frac{2d\sqrt{-\cot^2(e+fx)}\sqrt{\frac{b+a\csc(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2a}{a+b}\right)\sqrt{g\sin(e+fx)}\,\text{ta}}{c(c+d)fg\sqrt{a+b\sin(e+fx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5612 vs. $2(246) = 492$.

Time = 31.09 (sec) , antiderivative size = 5612, normalized size of antiderivative = 22.81

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/(Sqrt[g*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3342 vs. $2(227) = 454$.

Time = 2.86 (sec) , antiderivative size = 3343, normalized size of antiderivative = 13.59

method	result	size
default	Expression too large to display	3343

```
[In] int(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method
=_RETURNVERBOSE)
[Out] 1/f*(2*EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2
+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)
^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*
a*b*d*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)-EllipticPi((-(a*cot(f*x+e)-a*csc(f*
x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c
```

/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)
^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*c*d*(-a^2+b^2)^(1/2)+2*EllipticPi((-(a
*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b
+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*d^2*(-a^2+b^2)^(1/
2)+2*EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b
^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(
1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*
b*d*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)+EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+
e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(
c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(
1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*c*d*(-a^2+b^2)^(1/2)-2*EllipticPi((-(a*c
ot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(
-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b*d^2*(-a^2+b^2)^(1/2)
-4*EllipticF((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)
^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a
*b*d*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2)+4*EllipticF((-(a*cot(f*x+e)-a*csc(f*
x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+
b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*b^2*c*(-a^2+b^2)^(1/2)*(-c^2+d^2)^(1/2
)-EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)
^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2
)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3*d
*(-c^2+d^2)^(1/2)+2*EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2
)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)
-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(
1/2))^(1/2))*a*b^2*d*(-c^2+d^2)^(1/2)-EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+
e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(
c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(
1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3*d^2-EllipticPi((-(a*cot(f*x+e)-a*csc(f*x
+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/
(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^
(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b*c*d+2*EllipticPi((-(a*cot(f*x+e)-a*cs
c(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2
))*c/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+
b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2*d^2-EllipticPi((-(a*cot(f*x+e)-a
*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(
1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a
^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^3*d*(-c^2+d^2)^(1/2)+2*EllipticPi
((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/
2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c),1
/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2*d*(-c^2+d^2
)^(1/2)+EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^
2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2
)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))

```
*a^3*d^2+EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a
^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^
2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2)
)*a^2*b*c*d-2*EllipticPi((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(
b+(-a^2+b^2)^(1/2)))^(1/2),(b+(-a^2+b^2)^(1/2))*c/(c*(-a^2+b^2)^(1/2)+a*(-c
^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*a*b^2*d^2+2*EllipticF((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-
b)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)
^(1/2))^(1/2))*a^3*d*(-c^2+d^2)^(1/2)-2*EllipticF((-(a*cot(f*x+e)-a*csc(f*x
+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b
^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2*b*c*(-c^2+d^2)^(1/2)-4*EllipticF((-
(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2),
1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a*b^2*d*(-c^2+d^
2)^(1/2)+4*EllipticF((-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2)-b)/(b+(-
a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^
(1/2))*b^3*c*(-c^2+d^2)^(1/2))*(-(a*cot(f*x+e)-a*csc(f*x+e)-(-a^2+b^2)^(1/2
)-b)/(b+(-a^2+b^2)^(1/2)))^(1/2)*((a*cot(f*x+e)-a*csc(f*x+e)+(-a^2+b^2)^(1/
2)-b)/(-a^2+b^2)^(1/2))^(1/2)*((-csc(f*x+e)+cot(f*x+e))*a/(b+(-a^2+b^2)^(1/
2)))^(1/2)/(a+b*sin(f*x+e))^(1/2)*2^(1/2)*(1+cos(f*x+e))/(g*sin(f*x+e))^(1/
2)/a/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c)/(-c^2+d^2)^(1/2)/(c*(-
a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)-a*d+b*c)
```

Fricas [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sin(fx+e)+a}(d\sin(fx+e)+c)\sqrt{g\sin(fx+e)}}\,dx$$

```
[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(g*sin(f*x + e))/((b*c + a*d)*g*cos(
f*x + e)^2 - (b*c + a*d)*g + (b*d*g*cos(f*x + e)^2 - (a*c + b*d)*g)*sin(f*x
 + e)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \int \frac{1}{\sqrt{g\sin\left(e+fx\right)}\sqrt{a+b\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2), x)

[Out] Integral(1/(sqrt(g*sin(e + f*x))*sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))), x)

Giac [F]

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$= \int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}(d\sin\left(fx+e\right)+c)\sqrt{g\sin\left(fx+e\right)}}\,dx$$

[In] integrate(1/(c+d*sin(f*x+e))/(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{g\sin(e+fx)}\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))}\,dx$$
$$=\int \frac{1}{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{a+b\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

```
[In] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))),x)
[Out] int(1/((g*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x
))), x)
```

3.46 $\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal result 321
Rubi [A] (verified) 321
Mathematica [C] (warning: unable to verify) 323
Maple [C] (warning: unable to verify) 323
Fricas [F(-1)] 324
Sympy [F] 324
Maxima [F] 324
Giac [F] 324
Mupad [F(-1)] 325

Optimal result

Integrand size = 39, antiderivative size = 254

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$$
$$= \frac{2\sqrt{c+d}\sqrt{g}\sqrt{\frac{c(1-\csc(e+fx))}{c+d}}\sqrt{\frac{c(1+\csc(e+fx))}{c-d}}\,\text{EllipticPi}\left(\frac{c+d}{d},\arcsin\left(\frac{\sqrt{g}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{g\sin(e+fx)}}\right),-\frac{c+d}{c-d}\right)\tan(e+fx)}{bf}$$
$$+\frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{d+c\csc(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2c}{c+d}\right)\sqrt{g\sin(e+fx)}\,t}{b(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2*EllipticPi(g^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(g*sin(f*x+e))^(1/2
),(c+d)/d,((-c-d)/(c-d))^(1/2))*(c+d)^(1/2)*g^(1/2)*(c*(1-csc(f*x+e))/(c+d)
)^(1/2)*(c*(1+csc(f*x+e))/(c-d))^(1/2)*tan(f*x+e)/b/f+2*(-a*d+b*c)*Elliptic
Pi(1/2*(1-csc(f*x+e))^(1/2)*2^(1/2),2*a/(a+b),2^(1/2)*(c/(c+d))^(1/2))*(-co
t(f*x+e)^2)^(1/2)*((d+c*csc(f*x+e))/(c+d))^(1/2)*(g*sin(f*x+e))^(1/2)*tan(f
*x+e)/b/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used

$= \{3008, 2888, 3016\}$

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$$
$$= \frac{2(bc-ad)\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{c\csc(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right.\right.}{bf(a+b)\sqrt{c+d\sin(e+fx)}}$$
$$+\frac{2\sqrt{g}\sqrt{c+d}\tan(e+fx)\sqrt{\frac{c(1-\csc(e+fx))}{c+d}}\sqrt{\frac{c(\csc(e+fx)+1)}{c-d}}\,\text{EllipticPi}\left(\frac{c+d}{d},\arcsin\left(\frac{\sqrt{g}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{g\sin(e+fx)}}\right),-\frac{c+d}{c-d}\right)}{bf}$$

```
[In] Int[(Sqrt[g*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[c + d]*Sqrt[g]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*Sqrt[(c*(1 + Cs
c[e + f*x]))/(c - d)]*EllipticPi[(c + d)/d, ArcSin[(Sqrt[g]*Sqrt[c + d*Sin[
e + f*x]])/(Sqrt[c + d]*Sqrt[g*Sin[e + f*x]])], -((c + d)/(c - d))]*Tan[e +
 f*x])/(b*f) + (2*(b*c - a*d)*Sqrt[-Cot[e + f*x]^2]*Sqrt[(d + c*Csc[e + f*x
])/(c + d)]*EllipticPi[(2*a)/(a + b), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]
], (2*c)/(c + d)]*Sqrt[g*Sin[e + f*x]]*Tan[e + f*x])/(b*(a + b)*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
 d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
 d^2, 0] && PosQ[(c + d)/b]
```

Rule 3008

```
Int[(Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]])/((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[b/d, In
t[Sqrt[g*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/
d, Int[Sqrt[g*Sin[e + f*x]]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
```

```
d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{d \int \frac{\sqrt{g \sin(e+fx)}}{\sqrt{c+d \sin(e+fx)}} \, dx}{b} - \frac{(-bc+ad) \int \frac{\sqrt{g \sin(e+fx)}}{(a+b \sin(e+fx))\sqrt{c+d \sin(e+fx)}} \, dx}{b}$$

$$= \frac{2\sqrt{c+d}\sqrt{g}\sqrt{\frac{c(1-\csc(e+fx))}{c+d}}\sqrt{\frac{c(1+\csc(e+fx))}{c-d}} \, \text{EllipticPi}\left(\frac{c+d}{d}, \arcsin\left(\frac{\sqrt{g}\sqrt{c+d \sin(e+fx)}}{\sqrt{c+d}\sqrt{g \sin(e+fx)}}\right), -\frac{c+d}{c-d}\right) \tan(e +}{bf}$$

$$+ \frac{2(bc-ad)\sqrt{-\cot^2(e+fx)}\sqrt{\frac{d+c \csc(e+fx)}{c+d}} \, \text{EllipticPi}\left(\frac{2a}{a+b}, \arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right), \frac{2c}{c+d}\right) \sqrt{g \sin(e}{b(a+b)f\sqrt{c+d \sin(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.30 (sec) , antiderivative size = 23019, normalized size of antiderivative = 90.63

$$\int \frac{\sqrt{g \sin(e+fx)}\sqrt{c+d \sin(e+fx)}}{a+b \sin(e+fx)} \, dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[g*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x]),x]
```

```
[Out] Result too large to show
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 5489, normalized size of antiderivative = 21.61

method	result	size
default	Expression too large to display	5489

```
[In] int((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{g\sin{(e+fx)}}\sqrt{c+d\sin{(e+fx)}}}{a+b\sin{(e+fx)}}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))**(1/2)*(g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{d\sin{(fx+e)}+c}\sqrt{g\sin{(fx+e)}}}{b\sin{(fx+e)}+a}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))/(b*sin(f*x + e) + a
), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{d\sin{(fx+e)}+c}\sqrt{g\sin{(fx+e)}}}{b\sin{(fx+e)}+a}\,dx$$

```
[In] integrate((c+d*sin(f*x+e))^(1/2)*(g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*sqrt(g*sin(f*x + e))/(b*sin(f*x + e) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

```
[In] int(((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)
),x)
[Out] int(((g*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)
), x)
```

3.47 $\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result 326
Rubi [A] (verified) 326
Mathematica [F] 327
Maple [B] (warning: unable to verify) 327
Fricas [F(-1)] 329
Sympy [F] 329
Maxima [F] 329
Giac [F] 330
Mupad [F(-1)] 330

Optimal result

Integrand size = 39, antiderivative size = 114

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{-\cot^2(e+fx)}\sqrt{\frac{d+c\csc(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2c}{c+d}\right)\sqrt{g\sin(e+fx)}\tan(e+fx)}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2*EllipticPi(1/2*(1-csc(f*x+e))^(1/2)*2^(1/2),2*a/(a+b),2^(1/2)*(c/(c+d))^(
1/2))*(-cot(f*x+e)^2)^(1/2)*((d+c*csc(f*x+e))/(c+d))^(1/2)*(g*sin(f*x+e))^(
1/2)*tan(f*x+e)/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {3016}

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\tan(e+fx)\sqrt{-\cot^2(e+fx)}\sqrt{g\sin(e+fx)}\sqrt{\frac{c\csc(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

```
[In] Int[Sqrt[g*Sin[e + f*x]]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[-Cot[e + f*x]^2]*Sqrt[(d + c*Csc[e + f*x])/(c + d)]*EllipticPi[(2*a
)/(a + b), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*c)/(c + d)]*Sqrt[g*Si
n[e + f*x]]*Tan[e + f*x])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 3016

```
Int[Sqrt[(g_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[2*Sqrt[
-Cot[e + f*x]^2]*(Sqrt[g*Sin[e + f*x]]/(f*(c + d)*Cot[e + f*x]*Sqrt[a + b*S
in[e + f*x]]))*Sqrt[(b + a*Csc[e + f*x])/(a + b)]*EllipticPi[2*(c/(c + d)),
 ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], 2*(a/(a + b))], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0]
```

Rubi steps

integral

$$= \frac{2\sqrt{-\cot^2(e+fx)}\sqrt{\frac{d+c\csc(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\arcsin\left(\frac{\sqrt{1-\csc(e+fx)}}{\sqrt{2}}\right),\frac{2c}{c+d}\right)\sqrt{g\sin(e+fx)}\tan(e+fx)}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] Integrate[Sqrt[g*Sin[e + f*x]]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] Integrate[Sqrt[g*Sin[e + f*x]]/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(107) = 214$.

Time = 2.92 (sec) , antiderivative size = 2590, normalized size of antiderivative = 22.72

method	result	size
default	Expression too large to display	2590

```
[In] int((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^
2)^(1/2)+d))^(1/2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^
(1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*d
*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)+2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)
```

```
+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c
*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2
)+d)/(-c^2+d^2)^(1/2))^(1/2))*d*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)-EllipticP
i(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/
2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1
/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*c^2*(-a^2+b^2)^(1
/2)+2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2
)^(1/2)+d))^(1/2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(
1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*d^
2*(-a^2+b^2)^(1/2)-EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+
d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)-
a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1
/2))^(1/2))*c^2*(-a^2+b^2)^(1/2)+2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(
-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(
-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+
d)/(-c^2+d^2)^(1/2))^(1/2))*d^2*(-a^2+b^2)^(1/2)+EllipticPi(((c*csc(f*x+e)-
c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),((-c^2+d^2)^(1
/2)+d)*a/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^
2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*a*c*(-c^2+d^2)^(1/2)-2*EllipticPi(
((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2)
,((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1/2
*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*b*d*(-c^2+d^2)^(1/2
)-EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1
/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2
)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/2))^(1/2))*a*c*(
-c^2+d^2)^(1/2)+2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2)^(1/2)+d
)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2)^(1/2)-a
*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2+d^2)^(1/
2))^(1/2))*b*d*(-c^2+d^2)^(1/2)+EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^
2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2
+b^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(
-c^2+d^2)^(1/2))^(1/2))*a*c*d+EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+
d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b
^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c
^2+d^2)^(1/2))^(1/2))*b*c^2-2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+
d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b
^2)^(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c
^2+d^2)^(1/2))^(1/2))*b*d^2-EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^
2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^
2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^
2+d^2)^(1/2))^(1/2))*a*c*d-EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^2
)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^2
)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^2
+d^2)^(1/2))^(1/2))*b*c^2+2*EllipticPi(((c*csc(f*x+e)-c*cot(f*x+e)+(-c^2+d^
2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2),-((-c^2+d^2)^(1/2)+d)*a/(c*(-a^2+b^
```

```
2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c),1/2*2^(1/2)*(((-c^2+d^2)^(1/2)+d)/(-c^
2+d^2)^(1/2))^(1/2))*b*d^2)*(g*sin(f*x+e))^(1/2)*2^(1/2)*((c*csc(f*x+e)-c*c
ot(f*x+e)+(-c^2+d^2)^(1/2)+d)/((-c^2+d^2)^(1/2)+d))^(1/2)*(c/((-c^2+d^2)^(1
/2)+d)*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1/(-c^2+d^2)^(1/2)*(-c*csc(f*x+e)+c
*cot(f*x+e)+(-c^2+d^2)^(1/2)-d))^(1/2)/(c+d*sin(f*x+e))^(1/2)*(cot(f*x+e)+c
sc(f*x+e))*a/(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-a*d+b*c)/(c*(-a^2+b^2)^
(1/2)+a*(-c^2+d^2)^(1/2)+a*d-b*c)/(-a^2+b^2)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$$

```
[In] integrate((g*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(g*sin(e + f*x))/((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{g\sin(fx+e)}}{(b\sin(fx+e)+a)\sqrt{d\sin(fx+e)+c}}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c)), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{g\sin\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

```
[In] integrate((g*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(g*sin(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sin(e+fx)}}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{g\,\sin\left(e+f\,x\right)}}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((g*sin(e + f*x))^(1/2)/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)
), x)
```

3.48 $\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal result 331
Rubi [A] (verified) 332
Mathematica [A] (verified) 333
Maple [C] (warning: unable to verify) 334
Fricas [F(-1)] 334
Sympy [F] 334
Maxima [F] 335
Giac [F] 335
Mupad [F(-1)] 335

Optimal result

Integrand size = 35, antiderivative size = 391

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx =$$

$$-\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\cdots}}{\sqrt{a+b}f}$$

$$+\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\cdots}}{\sqrt{a+b}f}$$

```
[Out] -2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e
))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/f/(a+b)^(1/2
)+2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+
e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*
sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))
^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/f/(a+b)^(1/
2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3028, 2890, 3024}

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$$
$$= \frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\operatorname{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\right.\right.}{f\sqrt{a+b}}$$
$$-\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\operatorname{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\operatorname{arcsi}\right.}{f\sqrt{a+b}}$$

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*f) + (2*
Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c
+ d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/
((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
- d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*f)
```

Rule 2890

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 3024

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*((a + b*Sin[
e + f*x])/(c*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(-(b*c - a*d))*((
1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + S
in[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a +
 b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*S
```

```
in[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
]
```

Rule 3028

```
Int[(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]])/sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[d, Int[Sqrt[a +
 b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[c, Int[Sqrt[a + b*
Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] || NeQ[c^2 -
d^2, 0])
```

Rubi steps

$$\text{integral} = c\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx + d\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= -\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e}{(c+d)(a+b\sin(e}}}{\sqrt{a+b}f}$$

$$+\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e}{(c+d)(a+b\sin(e}}}{\sqrt{a+b}f}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.70

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx =$$

$$-\frac{2\sqrt{c+d}\left(\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) - \text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c}}{\sqrt{c+d}\sqrt{c}}\right.\right.\right.}{\sqrt{a+b}}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*(EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sq
rt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(
c + d))/((a + b)*(c - d))] - EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sq
rt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])]
, ((a - b)*(c + d))/((a + b)*(c - d))])*Sec[e + f*x]*Sqrt[((b*c - a*d)*(-1
+ Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin
[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a +
 b]*f)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.63 (sec) , antiderivative size = 242134, normalized size of antiderivative = 619.27

method	result	size
default	Expression too large to display	242134

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x,method=_RETU
RNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\int \frac{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{\sin(e+fx)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2)/sin(f*x+e),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/sin(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}}{\sin(fx+e)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x
)
```

Giac [F]

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{b\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}}{\sin(fx+e)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e),x, algor
ithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sin(f*x + e), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = \int \frac{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}{\sin(e+fx)}\,dx$$

```
[In] int(((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x),x)
[Out] int(((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2))/sin(e + f*x), x
)
```

3.49 $\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result 336
Rubi [A] (verified) 336
Mathematica [A] (verified) 337
Maple [B] (warning: unable to verify) 338
Fricas [F(-1)] 338
Sympy [F] 338
Maxima [F] 339
Giac [F] 339
Mupad [F(-1)] 339

Optimal result

Integrand size = 35, antiderivative size = 198

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc}{(c}}}{\sqrt{a+b}cf}$$

```
[Out] -2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e
))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/c/f/(a+b)^(1
/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3024}

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\text{arcsi}\right.}{cf\sqrt{a+b}}$$

```
[In] Int[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*c*f)
```

Rule 3024

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*((a + b*Sin[
e + f*x])/(c*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(-(b*c - a*d))*((
1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + S
in[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a +
 b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*S
in[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
]
```

Rubi steps

integral =

$$-\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\cdots}}{\sqrt{a+b}cf}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{\frac{(-bc+ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\cdots}}{\sqrt{a+b}cf}$$

```
[In] Integrate[(Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]],
x]
```

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((-(b*c) + a*d)*(1 - Sin[e + f*
x]))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*c*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 28725 vs. 2(183) = 366.

Time = 7.15 (sec) , antiderivative size = 28726, normalized size of antiderivative = 145.08

method	result	size
default	Expression too large to display	28726

```
[In] int((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}\sin(e+fx)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)),
x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(sqrt(d*sin(f*x + e) + c)*sin(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(sqrt(d*sin(f*x + e) + c)*sin(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(sin(e + f*x)*(c + d*sin(e + f*x))^(1/2)), x
)
```

3.50 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result 340
Rubi [A] (verified) 341
Mathematica [A] (verified) 342
Maple [B] (warning: unable to verify) 343
Fricas [F(-1)] 343
Sympy [F] 343
Maxima [F] 344
Giac [F] 344
Mupad [F(-1)] 344

Optimal result

Integrand size = 35, antiderivative size = 398

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc}{(c}}}{a\sqrt{a+b}cf} - \frac{2b\sqrt{a+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(1+}{(a-b)(c+d}}}{a\sqrt{c+d}(bc-ad)f}$$

```
[Out] -2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e
))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/a/c/f/(a+b)^
(1/2)-2*b*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin
(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))
*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/a/(-a*d+b*c)/f/(c+d)^(
1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3026, 2897, 3024}

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx =$$
$$-\frac{2b\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{af\sqrt{c+d}(bc-ad)}$$
$$-\frac{2\sqrt{c+d}\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\text{arc}\right.}{acf\sqrt{a+b}}$$

```
[In] Int[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (-2*Sqrt[c + d]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(a*Sqrt[a + b]*c*f) -
 (2*b*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(
Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))
]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f
*x])))]*(c + d*Sin[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2897

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 3024

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/(sin[(e_.) + (f_.)*(x_)]*Sqr
t[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*((a + b*Sin[
e + f*x])/(c*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(-(b*c - a*d))*((
1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + S
in[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a +
 b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*S
```

```
in[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
]
```

Rule 3026

```
Int[1/(sin[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*S
qrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-b/a, Int[1/(
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[1/a, Int[
Sqrt[a + b*Sin[e + f*x]]/(Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (NeQ[a^2 - b^2, 0] ||
 NeQ[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\csc(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{2\sqrt{c+d}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+\dots}{(c+d)(a+b\sin(e+\dots}}}{a\sqrt{a+b}cf}$$

$$-\frac{2b\sqrt{a+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\dots}}{a\sqrt{c+d}(bc-ad)f}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.94

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$

$$= \frac{2\sec(e+fx)\left(-\frac{(c+d)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(-1+\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(1+\sin(e+fx))}{(c-d)(a+b\sin(e+fx))}}(a+b\dots}{c}\dots\right.}{a\sqrt{a+\dots}}$$

```
[In] Integrate[Csc[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x]
```

```
[Out] (2*Sec[e + f*x]*(-(((c + d)*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
 ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[((b*c - a*d)*(-1 + Sin[e + f*x])
)/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/c) - (b*(a + b)*Elliptic
```

```
F[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((-(b*c) + a*d)*(-1
 + Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]))]*(c + d*Sin[e + f*x]))/(b*c
- a*d)))/(a*Sqrt[a + b]*Sqrt[c + d]*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 24289 vs. $2(368) = 736$.

Time = 7.12 (sec) , antiderivative size = 24290, normalized size of antiderivative = 61.03

method	result	size
default	Expression too large to display	24290

```
[In] int(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\sin(e+fx)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))*sin(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$
$$=\int \frac{1}{\sqrt{b\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)
), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$
$$=\int \frac{1}{\sqrt{b\sin(fx+e)+a}\sqrt{d\sin(fx+e)+c}\sin(fx+e)}\,dx$$

```
[In] integrate(1/sin(f*x+e)/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sin(f*x + e)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$$
$$=\int \frac{1}{\sin(e+fx)\,\sqrt{a+b\,\sin(e+fx)}\,\sqrt{c+d\,\sin(e+fx)}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),
 x)
```

3.51 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$

Optimal result 345
Rubi [A] (verified) 345
Mathematica [F] 347
Maple [F] 347
Fricas [F] 347
Sympy [F(-1)] 348
Maxima [F] 348
Giac [F] 348
Mupad [F(-1)] 349

Optimal result

Integrand size = 38, antiderivative size = 157

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$=\frac{2^{\frac{1}{2}+n}\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,-p,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{B(1+\sin(e+fx))}{A-B}\right)\sec(e+fx)(1-\sin(e+f}{af(1+2m)}$$

```
[Out] 2^(1/2+n)*AppellF1(1/2+m,-p,1/2-n,3/2+m,-B*(1+sin(f*x+e))/(A-B),1/2+1/2*sin
(f*x+e))*sec(f*x+e)*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^(1+m)*(A+B*sin(
f*x+e))^p*(c-c*sin(f*x+e))^n/a/f/(1+2*m)/(((A+B*sin(f*x+e))/(A-B))^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3087, 145, 144, 143}

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$=\frac{2^{n+\frac{1}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^n(A+B\sin(e+fx))^p\,\Big(}{af(2m+1)}$$

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])^p*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*AppellF1[1/2 + m, 1/2 - n, -p, 3/2 + m, (1 + Sin[e + f*x])/2,
-((B*(1 + Sin[e + f*x]))/(A - B))]*Sec[e + f*x]*(1 - Sin[e + f*x])^(1/2 - n
```

```
)*(a + a*Sin[e + f*x])^(1 + m)*(A + B*Sin[e + f*x])^p*(c - c*Sin[e + f*x])^
n)/(a*f*(1 + 2*m)*((A + B*Sin[e + f*x])/(A - B))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\right)\mathrm{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(A+Bx)^p(c-cx)^{-\frac{1}{2}+n}\,dx,x,\right.}{f}$$

$$= \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))^p\left(\frac{a(A+B\sin(e+fx))}{aA-aB}\right)^{-p}\sqrt{c-c\sin(e+fx)}\right) \mathrm{S}}{f}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}\sec(e+fx)\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))^p\left(\frac{a(A+B\sin(e+fx))}{aA-aB}\right)^{-p}(c-c\sin(e+f\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n}\,\mathrm{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,-p,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)),-\frac{B(1+\sin(e+fx))}{A-B}\right)\sec(e+fx)(1-\sin}{af(1+}$$

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$= \int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])^p*(c - c*Sin[e + f*x]
)^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])^p*(c - c*Sin[e + f*x]
)^n, x]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m\,(A+B\sin(fx+e))^p\,(c-c\sin(fx+e))^n\,dx$$

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))^p*(c-c*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))^p*(c-c*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)^p(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))^p*(c-c*sin(f*x+e))^n,x, algor
ithm="fricas")
[Out] integral((B*sin(f*x + e) + A)^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))**p*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)^p(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))^p*(c-c*sin(f*x+e))^n,x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))^p(c-c\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)^p(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))^p*(c-c*sin(f*x+e))^n,x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))^p (c - c\sin(e + fx))^n \, dx$$
$$= \int (A + B\,\sin(e + f\,x))^p\,(a + a\,\sin(e + f\,x))^m\,(c - c\,\sin(e + f\,x))^n \,dx$$

```
[In] int((A + B*sin(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
[Out] int((A + B*sin(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x
)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 351

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
      print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```